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AFFDL-TR-73-155  
PART II

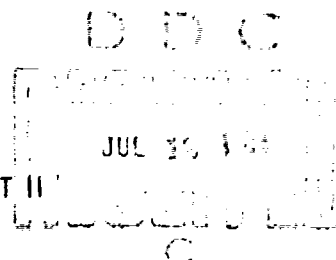
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# ACOUSTIC FATIGUE DESIGN CRITERIA FOR ELEVATED TEMPERATURES

CECIL W. SCHNEIDER

LOCKHEED-GEORGIA COMPANY

TECHNICAL REPORT AFFDL-TR-73-155, PART II



MARCH 1974

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# **ACOUSTIC FATIGUE DESIGN CRITERIA FOR ELEVATED TEMPERATURES**

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## FOREWORD

This report was prepared by the Lockheed-Georgia Company, Marietta, Georgia, for the Aero-Acoustics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract F33615-72-C-1141. The work described herein is a continuing part of the Air Force Systems Command's exploratory development program to establish tolerance levels and design criteria for acoustic fatigue prevention for flight vehicles. The work was directed under Project 1471, "Aero-Acoustic Problems in Air Force Flight Vehicles," Task 147101, "Sonic Fatigue". Mr. Davey L. Smith (AFFDL/FYA) was the Task Engineer.

This report concludes the work on Contract F33615-72-C-1141, which covered a period from April 1972 to December 1973. This report is one of two issued under this contract; AFFDL-TR-73-155, Part I, contains a complete description of the program and the results. The Lockheed-Georgia Company report identification for this AFFDL document is LG73ER0183.

The manuscript was submitted by the author 31 October 1973 for Publication. This technical report has been reviewed and is approved.

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## ABSTRACT

An analytical and experimental program was conducted to develop acoustic fatigue design criteria for aircraft structures subjected to intense noise in a high temperature environment. Equations for the dynamic response of a buckled panel were formulated for simply supported boundary conditions using large deflection plate theory. Random amplitude acoustic fatigue testing of representative aircraft structure was accomplished at temperatures up to 600°F to provide data for correlation with the analytical results. Empirical design criteria were developed in the form of equations and nomographs for predicting the thermal and dynamic response of aircraft structures subjected to combined environments. The empirical design criteria are presented in handbook format for design use; examples and computer programs are also presented.

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## LIST OF SYMBOLS

AR	Aspect ratio function = $3(b/a)^2 + 3(a/b)^2 + 2$
$a, a_i$	Dimension of panel in the x-direction - inch
$b, b_i$	Dimension of panel in the y-direction - inch
d	Stiffener height - inch
E	Modulus of elasticity - psi
$E_0$	Modulus of elasticity at ambient temperature - psi
F <sub>11</sub>	Aspect ratio function = $(b/a) + (a/b)$
f	Frequency - Hz
$f_0$	Room temperature fundamental mode response frequency - Hz
h	Skin thickness - inch
$I_{xx}, I_{yy}, \text{etc.}$	Cross-section second area moment of inertia of stiffening member - in <sup>4</sup>
$I^*$	Cross-section moment of inertia of stiffening member, see Equation (7) - in <sup>4</sup>
$N_x, N_y, N_{xy}$	In-plane skin loading - lb/in
N	Cycles to failure
R	Aspect ratio function, see Equation (11)
r	Temperature ratio = $T/T_c$
$S_L$	Sound pressure spectrum level - dB
T	Temperature increase - °F above ambient
$T_c$	Critical buckling temperature of skin - °F above ambient
$u, v, w$	In-plane displacements in x, y and z-directions
$W_0$	Skin thermal buckling amplitude of center bay of multi-bay panel - inch

# LIST OF SYMBOLS (CONT'D)

$W_{ij}$	Total transverse displacement of vibrating panel
$w, y, z$	Coordinate direction
$\alpha$	Coefficient of thermal expansion - (in/in)/°F
$\nu$	Mass density - lb-sec <sup>2</sup> /in <sup>4</sup>
$\zeta$	Damping ratio
$\mu$	Micro - 10 <sup>-6</sup>
$\nu$	Poisson's ratio
$\pi$	3.1415927
$\sigma_m$	Mean stress - psi (or ksi)
$\sigma_T$	Thermal expansion stress - psi (or ksi)
$\sigma_x, \sigma_y$	Normal stress in the x and y-directions - psi (or ksi)
$\sigma_{xb}, \sigma_{yb}$	Thermal buckling stress in x and y-directions - psi (or ksi)
$\tilde{\sigma}_x, \tilde{\sigma}_y$	Dynamic stress in x and y-directions - psi (or ksi) rms
$\tau_{xy}$	Shear stress in the x-y plane - psi
$\hat{p}(f)$	Acoustic pressure density - psi/√ Hz

## ABBREVIATIONS

dB	Decibel (Re: 0.0002 microbar)
F	Fahrenheit
ksi	1000 psi
psi	lb/in <sup>2</sup>
rms	Root Mean Square
RT	Room Temperature
SPL	Sound Pressure Level

## I - INTRODUCTION

A number of investigations have been conducted in recent years to develop and refine acoustic fatigue design criteria for aircraft structures. Some of these investigations have refined the design criteria for stiffened-skin structures, and have helped to reduce the uncertainty involved in predicting dynamic response and fatigue characteristics of these structures when subjected to propulsion system or aerodynamic noise at ambient temperatures. However, when unusual structural configurations or environmental conditions are encountered, the existing design methods become less applicable and the judgment of the design engineer must be relied upon more heavily. Structural applications are commonly encountered today in the near noise field of an operating turbojet or turbofan engine where severe noise, high temperatures, static loading, and vibratory buffet occur simultaneously or in conjunction with each other.

This program extends the basic design technology for ambient temperature aluminum structures to include the effects of simultaneous application of thermal and acoustic environments. A complete and detailed description of this program is presented in AFFDL-TR-73-155, PART I, including an analytical development for the dynamic response of heated structures before and after thermal buckling. The primary purpose of the analytical effort was to identify the parameters which describe the structural response; then the data requirements of the experimental program were defined in detail. Measured data were correlated with the analytical results to establish empirical design criteria in the form of equations, nomographs, and computer programs.

The significant results of this investigation are abstracted in this document as an aid to the aircraft designer. The design methodology is presented in a convenient, abbreviated format to simplify routine use. The governing assumptions and conditions are discussed in Section II.

The following sections delineate three available methods of application:

Section III - Design Relations for Manual Calculation

Section IV - Design Nomographs for Graphical Calculation

Section V - Computer Programs

The limiting conditions to the design methods are discussed in section VI to give the design engineer a quick guide as to the applicability of these methods to a particular design. Appendix I summarizes substructure section properties as an aid in the substructure design.

## II - ASSUMPTIONS AND CONDITIONS

The analytical development of AFFDL-TR-73-155, Part I, is based on certain simplifying assumptions. Many of these initial assumptions were negated by the development of the empirical relations. The significant remaining assumptions are that:

- o The temperature over the surface of a panel bay is uniform.
- o The substructure is temperature-independent.
- o The structural exciting force is random amplitude broad-band noise with a Gaussian distribution of amplitudes.
- o Dominant structural response occurs only in the fundamental mode of the panel.

The first of these must be closely approximated in practice, or unsymmetric buckling modes will occur and the empirical relations will not be valid. The second condition implies short time duration and localized exposure to the thermal environment, so that the substructure remains at a considerably lower temperature than the skin. The third condition implies that the design criteria are applicable to aircraft powered by jet engines, and are not applicable to aircraft powered by reciprocating or turboprop engines. The last of these conditions restricts the designs to consideration of single mode response which can prove erroneous for structures which exhibit significant multi-mode response.

The panel size, nomenclature, and sign convention for the design equations are shown in Figure 1. This model represents a single bay of a multi-bay stiffened-skin structure. Figure 2 is representative of a continuous multi-bay structure, where the model in Figure 1 represents the center bay. Throughout this handbook, the simplification  $a = a_2$ ,  $b = b_2$  will be used to describe the center bay size.

The center bay (or bay of interest) is shown in Figure 3 isolated from the remainder of the structure. The height of the stiffening member along the long side of the panel bay is defined as  $d$ . Throughout this report the panel bay aspect ratio is assumed to be greater than one ( $b/a \geq 1$ ).

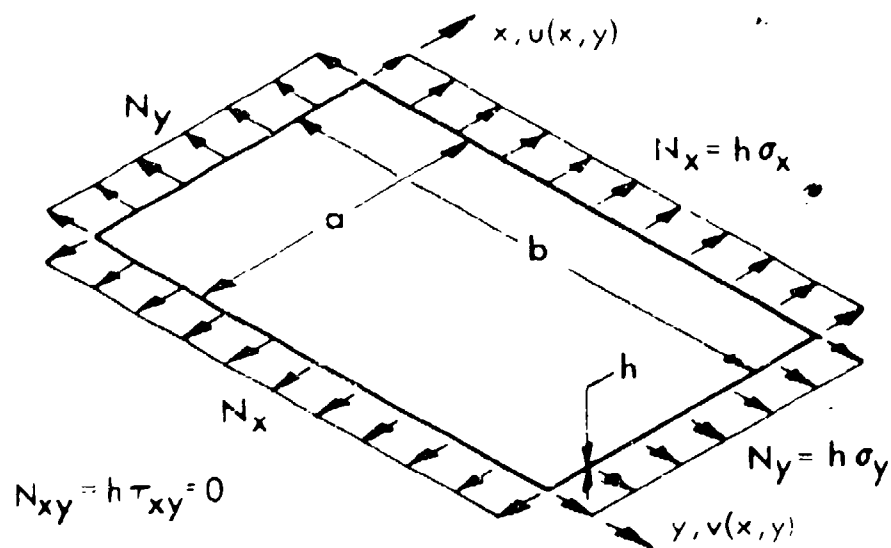
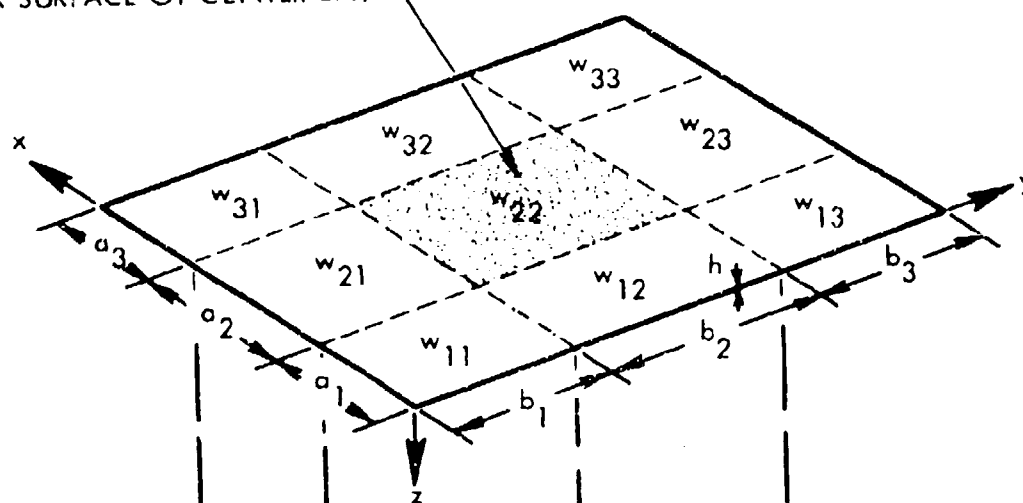


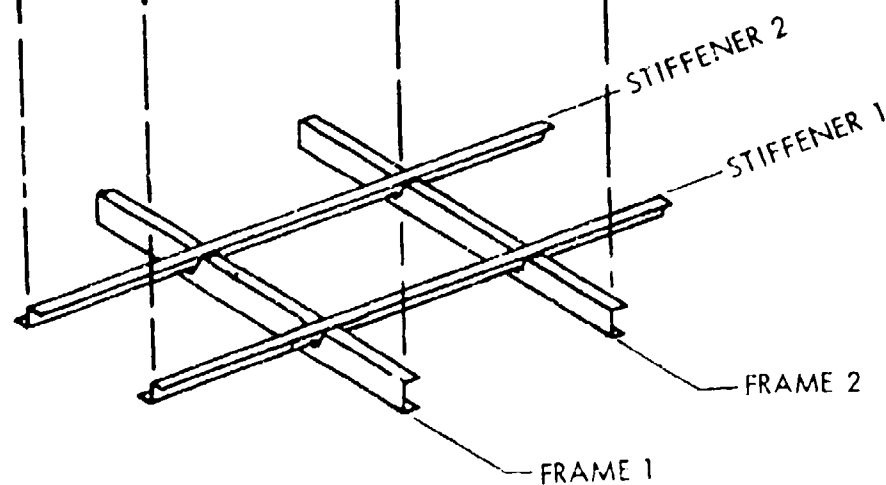
FIGURE 1. SINGLE BAY GEOMETRY AND NOTATION.



UNIFORM TEMPERATURE,  $T$   
OVER SURFACE OF CENTER BAY



a) PLATE GEOMETRY



b) SUPPORT STRUCTURE GEOMETRY

FIGURE 2. MULTI-BAY FLAT STIFFENED PANEL CONFIGURATION

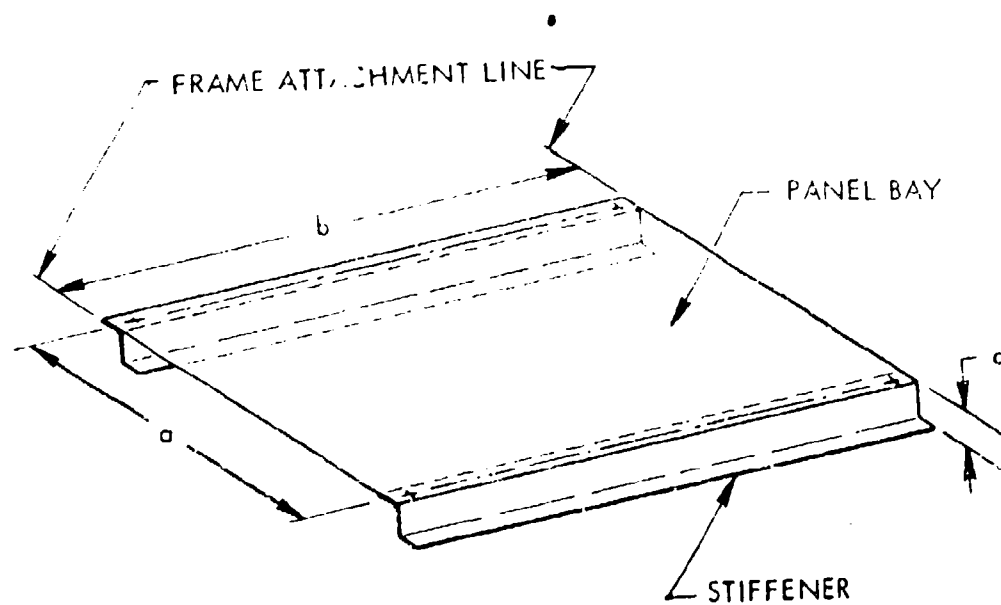


FIGURE 3. STIFFENED PANEL BAY

### III - DESIGN RELATIONS

Empirical design relations for ambient and elevated temperatures are presented in this section. Ambient temperature design criteria for aluminum structures are from AFFDL-TR-67-156<sup>2</sup> and AFFDL-TR-71-107<sup>3</sup>; elevated temperature design criteria are from AFFDL-TR-73-155, PART I. The source of design equations and information, other than Reference 1, will be acknowledged in this and the following sections by a reference in parenthesis. No attempt is made to cross-reference equation or figure numbers. All unreferenced criteria are from AFFDL-TR-73-155, PART I.

#### A. Ambient Temperature Design Criteria

The design criteria for ambient temperatures are unchanged from existing criteria. Only the dynamic response of the structure is involved in the design so long as the ambient temperature state does not cause buckling of the skin.

##### 1. Skin Design

The skin design criteria of AFFDL-TR-67-156 are valid for aluminum structures at ambient temperatures. The dynamic stresses at the midpoint of the long side will be greater than those at the midpoint of the short side if the panel responds in the fundamental mode. The dynamic stress at this point is (Reference 2):

$$\tilde{\sigma}_x = 1.62 \times 10^{-4} \left[ \frac{E}{Y} \right]^{1/4} \frac{\sigma^{1.25}}{h^{1.75}} \frac{\xi(f)}{\xi} \frac{(b/a)^{1.75}}{(AR)^{.84}} \quad - \text{ksi}_{\text{rms}} \quad (1)$$

Dynamic stresses at this point are also given by

$$\tilde{\sigma}_x = 3.60 \times 10^{-4} \left( \frac{b}{h} \right)^2 \frac{\xi(f)}{AR} \left[ \frac{f_o}{\xi} \right]^{1/2} \quad - \text{ksi}_{\text{rms}} \quad (2)$$

The fundamental mode frequency for a single bay of a multi-bay structure is given by

$$f_o = \frac{0.79 F_{11} h}{ab} \left[ \frac{E}{\nu(1 - \nu^2)} \right]^{1/2} \quad - \text{Hz} \quad (3)$$

The aspect ratio parameters are defined as

$$F_{11} = b/a + a/b \quad (4a)$$

$$AR = 3(b/a)^2 + 3(a/b)^2 + 2 \quad (4b)$$

while the acoustic pressure density of the fundamental mode is

$$\Phi(f) = 2.91 \times 10^{[(S_L/20)-9]} - \text{psi}/\sqrt{\text{Hz}} \quad (5)$$

Comparison of these equations shows that, for identical configurations, Equation (1) gives higher stresses than (2). Since both are based on the same analytical model, the difference is in the data on which these empirical relations are based. These empirical relations can be considered as bounds for predicting dynamic stresses. The form of the latter equation lends itself to much easier solution.

Dynamic stresses calculated by these equations are used with suitable random amplitude fatigue curves to estimate fatigue life. Figure 4 contains random amplitude, reversed bending fatigue curves for 7075-T6 aluminum and 6Al-4V annealed titanium at ambient temperature. These curves were developed from coupon fatigue tests, and the riveted fatigue data are representative of typical riveted, stiffened-skin structure.

**EXAMPLE:** A flat aluminum alloy stiffened structure is to be designed to withstand an estimated sound pressure spectrum level of 120 dB for  $5 \times 10^7$  cycles at ambient temperature. The skin design is determined by the procedure described below.

Assume:

- o Damping ratio:  $\zeta = 0.012$
- o Panel width:  $a = 4.75$  inches
- o Aspect ratio:  $b/a = 1.5$  ( $AR = 10.08$ )

From Figure 4, the dynamic stress for riveted aluminum (mean value fatigue curve) corresponding to a design life of  $5 \times 10^7$  cycles is 3.8 ksi rms. Using Equation (1), the minimum skin gage is found by

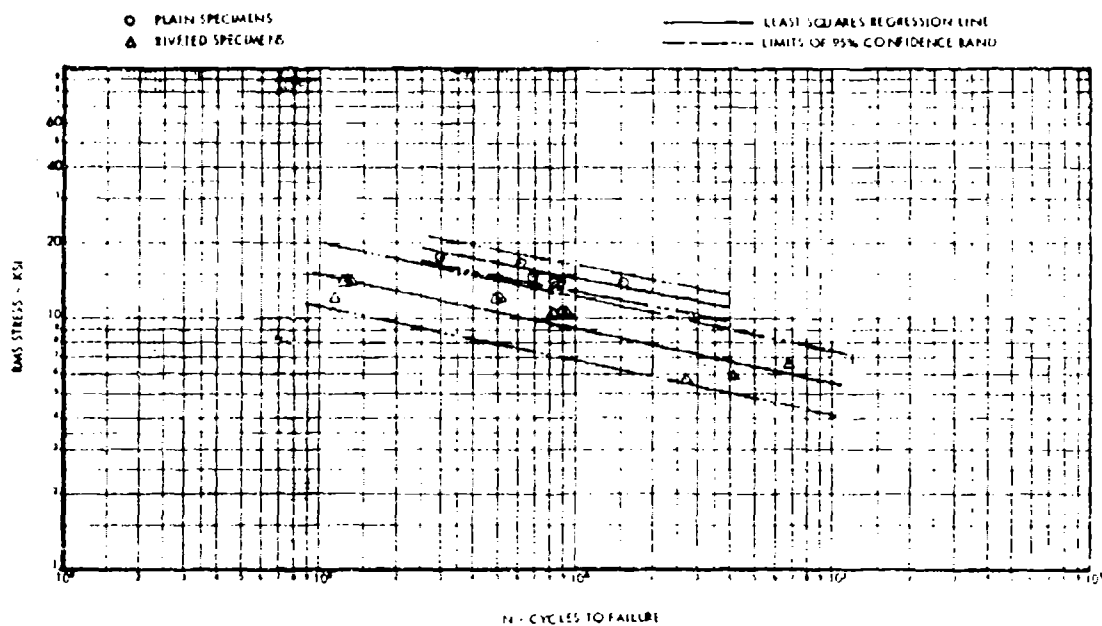
$$h^{1.75} = 1.62 \times 10^{-4} \left[ \frac{E}{Y} \right]^{1/4} \frac{a^{1.25}}{\tilde{\sigma}_x} \frac{\Phi(f)}{\zeta^{.56}} \frac{(b/a)^{1.75}}{(AR)^{.84}} - \text{in}$$

or

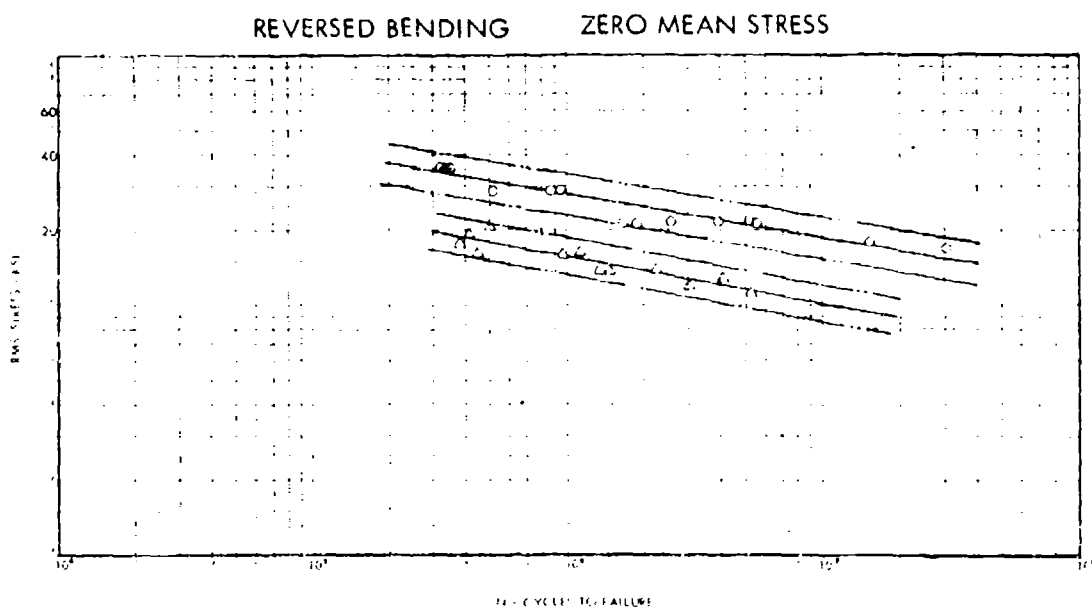
$$h = \left\{ 1.62 \times 10^{-4} \left[ \frac{10.3 \times 10^6}{2.66 \times 10^{-4}} \right]^{1/4} \frac{(4.75)^{1.25}}{(3.8)} \frac{(2.9 \times 10^{-3})}{(.012)^{.56} (10.08)^{.84}} \right\}^{.57} \quad (1.5)$$

$$= 0.023 \text{ in}$$

From Equation (3), the fundamental mode frequency is



a) ALUMINUM ALLOY 7075-T6



b) TITANIUM ALLOY 6Al-4V ANNEALED

FIGURE 4. RANDOM AMPLITUDE LOADING FATIGUE CURVES  
 AMBIENT TEMPERATURE

$$f_o = \frac{(0.79)(2.17)}{(4.75)(7.125)} \left[ \frac{10.3 \times 10^6}{(2.66 \times 10^{-4})(.8976)} \right]^{1/2} h = 1.052 \times 10^4 h$$

Substituting this into Equation (2) gives

$$h = \left\{ 3.60 \times 10^{-4} \frac{b^2}{\sigma_x} \frac{\Phi(f)}{AR} \left[ \frac{1.052 \times 10^4}{\zeta} \right]^{1/2} \right\}^{2/3} - \text{in}$$

$$= \left\{ \frac{3.60 \times 10^{-4} (7.125)^2 (2.9 \times 10^{-3})}{(3.8)(10.08)} \left[ \frac{1.052 \times 10^4}{.012} \right]^{1/2} \right\}^{2/3}$$

$$= 0.013 \text{ in.}$$

The thicker of the two skin thicknesses should be selected if skin failure is to be completely avoided.

## 2. Stiffener Flange Design

The acoustic loading on the surface of a stiffened panel is transferred to the substructure predominately by a transverse shear loading, causing the open-section stiffeners to bend and twist. The stiffener loading is reacted along the skin-stiffener attachment (rivet) line and at the clip attachments to the frames. The resulting flange stress is given by (Reference 3),

$$\tilde{\sigma}_f = 0.9 \left[ \frac{0.0121 b^3 d}{I^* F_{11}} \Phi(f) \left( \frac{f_o}{\zeta} \right)^{1/2} \right]^{1/5} - \text{ksi}_{\text{rms}} \quad (6)$$

where

$$I^* = (I_{xx} I_{zz} - I_{xz}^2) / I_{zz} \quad (7)$$

This relation is valid only for the fundamental mode of the panel.

The above flange stress is used in conjunction with a fatigue curve developed for flange failures and presented in Figure 5 (Reference 3).

**EXAMPLE:** A flat aluminum alloy stiffened structure is to be designed to withstand an estimated sound pressure spectrum level of 120 dB for  $5 \times 10^7$  cycles at ambient temperature. The stiffener design is determined by the procedure described below.

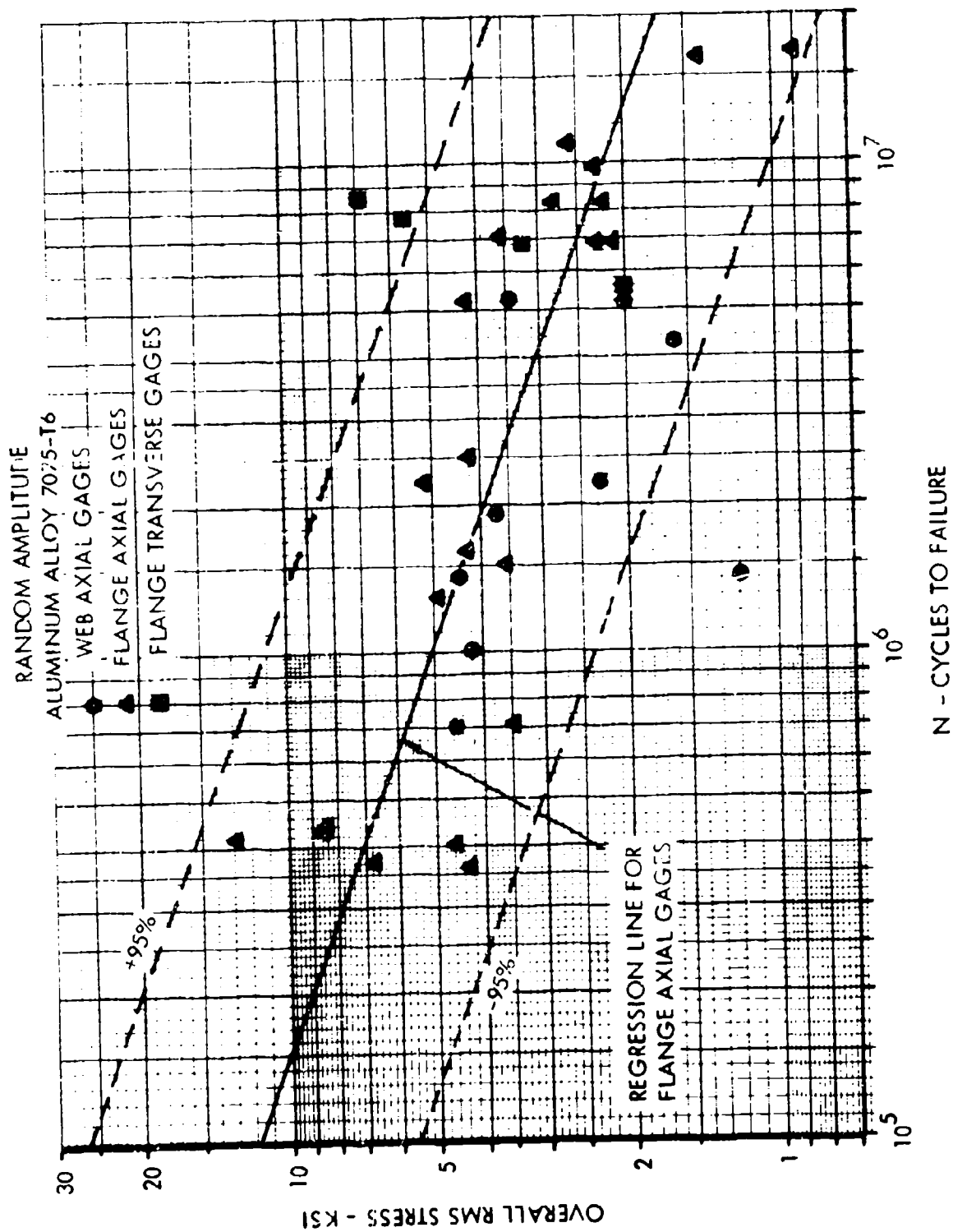


FIGURE 5. STIFFENED PANEL STIFFENER FATIGUE CURVE  
 AMBIENT TEMPERATURE: (FROM REFERENCE 3)

Assume:

- o Damping ratio:  $\zeta = 0.012$
- o Panel width:  $a = 4.75$  inches
- o Aspect ratio:  $b/a = 1.5$  ( $F_{11} = 2.17$ )

From the previous example, the skin thickness is taken as  $h = 0.025$  inch, for which the fundamental mode frequency is

$$f_o = 1.052 \times 10^4 \times 0.025$$

$$= 263 \text{ Hz}$$

The stiffener is a zee section 0.040 inch thick with a flange width of 0.75 inch and height of 1.25 inch. The section properties give a value of  $I^* = 0.01255 \text{ inch}^4$ . (See Appendix I for section property calculations.)

From Equation (6), the flange stress is

$$\tilde{\sigma}_f = 0.9 \left[ \frac{0.0121(7.125)^3(1.25)(2.9 \times 10^{-3})}{(0.01255)(2.17)} \left[ \frac{263}{.012} \right]^{1/2} \right]^{1/5}$$

$$= 2.19 \text{ ksi}_{\text{rms}}$$

From Figure 5, the life is estimated to be  $N = 1.0 \times 10^7$  cycles, or slightly less than the design requirement.

The above procedure should then be repeated using a thicker or deeper zee stiffener until the desired life is achieved. It is also possible to reduce the stiffener spacing, thereby increasing the fundamental frequency and perhaps altering the excitation (dependent on the excitation spectrum shape). However, it must be remembered that changing the spacing or skin thickness alters the skin stress, resulting in a conservative skin design.

### 3. Damping of Stiffened-Skin Structures

The estimation of a damping value for structural design is normally based on previous test data for similar structure and the experience and judgment of the design engineer. To aid the individual designer in this task, the damping data from this and other programs<sup>2-5</sup> were plotted versus frequency, and the resulting plot is presented in Figure 6. All of the data, except those from Reference 4, are for fundamental mode response of stiffened-skin structure. Although the data scatter is such that no definitive estimation criteria are derived, some general observations may be drawn:



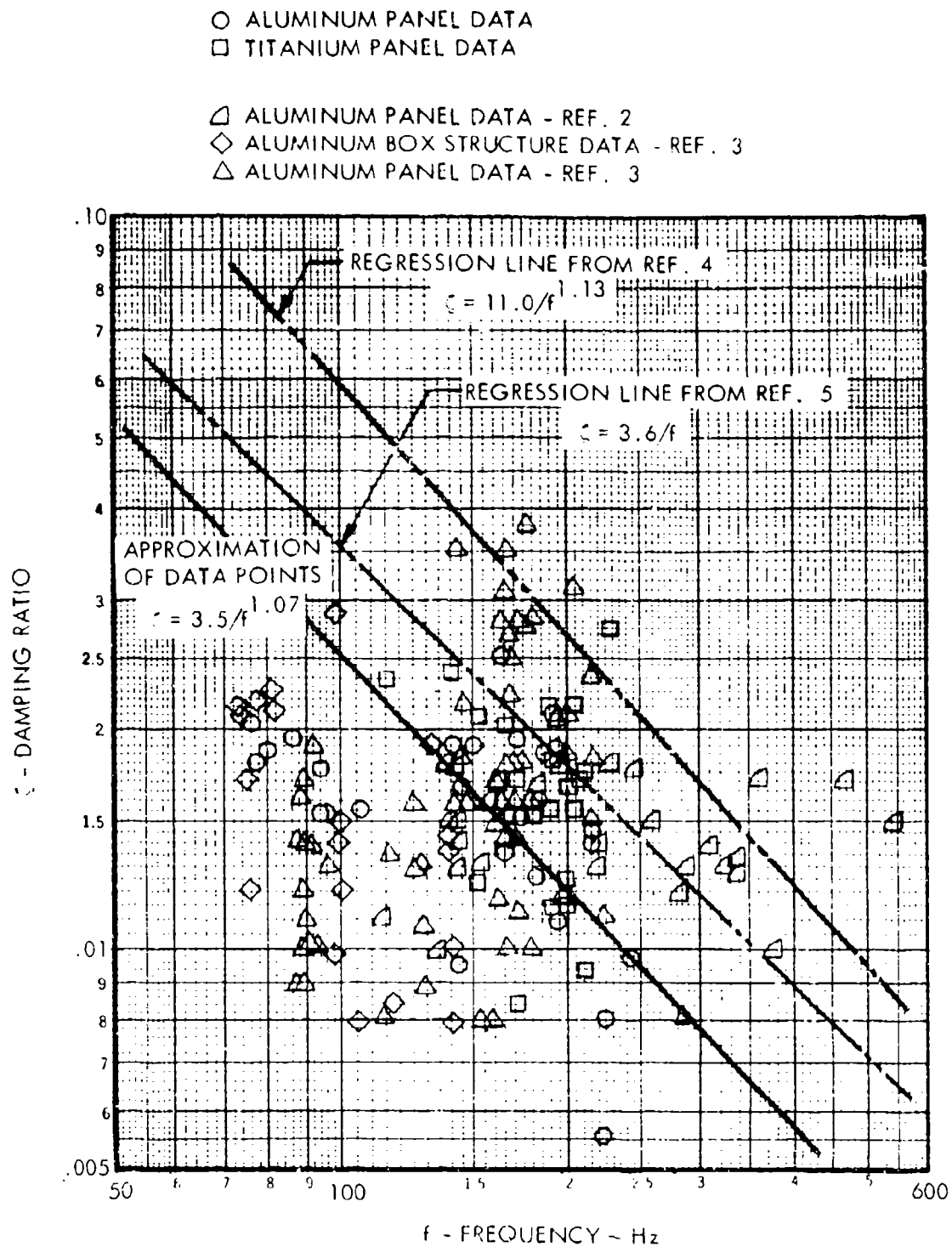


FIGURE 6. DAMPING VARIATION WITH FREQUENCY

- o The data of Reference 4, which included response of a single panel bay through the (3,3) mode, were modeled by an equivalent viscous damping, whereby the damping ratio is inversely proportional to the forcing frequency. The data were thus represented by an equation of the form

$$\zeta = 11.0/f^{1.13}$$

A similar curve was established by Reference 5;

$$\zeta = 3.6/f$$

Both of these representations are shown in the curve of Figure 6.

- o A similar approximation was established through the data points of Figure 6, with the slope taken as an average of the Reference 4 and 5 slopes. The equation of this approximation is

$$\zeta = 3.5/f^{1.07}$$

- o The mean value, for the fundamental mode, of all damping data shown in Figure 6 is

$$\zeta_{\text{mean}} = 0.016$$

This mean value can be used in lieu of definitive test data for a particular configuration.

- o The upper and lower bounds of all measured data are  $\zeta_U = 0.038$  and  $\zeta_L = 0.0056$ . Application of these bounds on damping ratio will provide limiting values for dynamic stresses.

The foregoing may be useful for rough estimates of structural damping. The actual choice of damping ratio is left to the judgment of the design engineer, with the above offered as a guide.

## 8. Elevated Temperature Design Criteria

The elevated temperature design criteria are used in essentially the same sequence in which they are discussed in the following subsections. Since more than one method of application is available, all the criteria will be summarized and then followed by examples of their use.

### 1. Skin Buckling Temperature

The critical buckling temperature of a single panel, such as that shown in Figure 3, is defined as the temperature increase at which the skin buckles and is given by

$$T_c = \frac{5.25 h^2 F_{11}}{a b (1 + \nu)} \quad - ^\circ F \text{ above ambient} \quad (8)$$

The temperature ratio is then defined as

$$r = T/T_c \quad (9)$$

where T is the temperature rise of the structure above ambient.

### 2. Skin Buckling Amplitude

The maximum skin buckling amplitude, at the midpoint of the panel bay shown in Figure 3, is given by

$$W_o = 2.50 h F_{11}^{1.75} \left[ \frac{r-1}{R} \right]^{1/2} \quad - \text{ inches} \quad (10)$$

where R is an aspect ratio parameter defined as

$$R = 3[(5 - \nu^2)F_{11}^2 - 2(5 + \nu)(1 - \nu)] \quad (11)$$

### 3. Thermal Stress

Thermal stresses due to in-plane expansion and skin buckling are given by the following equations for the midpoints of each side:

- o Midpoint of the panel bay long side ( $y = b/2$ )

$$\sigma_x = \left\{ -\frac{E \alpha T}{1 - \nu} + \frac{0.82 E W_o^2}{a b (1 - \nu^2)} \left[ \frac{b}{a} (2 - \nu^2) + \nu \frac{a}{b} \right] \right\} 10^{-3} \quad - \text{ ksi} \quad (12)$$

- o Midpoint of the panel bay short side ( $x = a/2$ )

$$\sigma_y = \left\{ -\frac{E \alpha T}{1 - \nu} + \frac{1.66 E W_o^2}{a b (1 - \nu^2)} \left[ \frac{a}{b} (2 - \nu^2) + \nu \frac{b}{a} \right] \right\} 10^{-3} \quad - \text{ ksi} \quad (13)$$

Thermal stresses must be computed at the midpoint of both sides, since the short side stress is usually greater than that at the center of the long side. This is opposite to the magnitudes of the dynamic stresses at the two locations. The above relations can be simplified to

$$\sigma_x = \sigma_T + \sigma_{x_b} ; \quad \sigma_y = \sigma_T + \sigma_{y_b} \quad (14)$$

where the respective parameters are the in-plane expansion and buckling stresses as inferred from Equations (12) and (13).

#### 4. Elevated Temperature Frequency Response

The fundamental mode frequency at a temperature increase,  $T$ , is given by the following equations for the indicated temperature ranges:

$$\begin{aligned} f(r) &= f_0 [0.60 + 0.40(1 - r)^{1/2}] & - \text{Hz} & \quad (0 \leq r \leq 1) \\ &= f_0 [0.60 + 0.44(r - 1)^{1/2}] & - \text{Hz} & \quad (r \geq 1) \end{aligned} \quad (15)$$

#### 5. Dynamic Stress

Dynamic stresses at any temperature can be computed by the following equations for the indicated locations:

- o Rivet row at midpoint of long side

$$\tilde{\sigma}_x = 3.60 \times 10^{-4} \left( \frac{b}{h} \right)^2 \frac{\Phi(f)}{AR} \left[ \frac{f(r)}{\zeta} \right]^{1/2} \quad - \text{ksi}_{\text{rms}} \quad (16)$$

- o Rivet row at midpoint of short side

$$\tilde{\sigma}_y = 13.0 \times 10^{-4} \left( \frac{a}{h} \right)^2 \frac{\Phi(f)}{AR} \left[ \frac{f(r)}{\zeta} \right]^{1/2} \quad - \text{ksi}_{\text{rms}} \quad (17)$$

The elevated temperature panel response frequency,  $f(r)$ , must be used for these computations. The stresses at both locations must generally be calculated for elevated temperature applications because of the interaction of the thermal and dynamic stresses.

**EXAMPLE:** A flat structure is to be designed for a service life of 100 hours at a sound pressure spectrum level of 140 dB and a service temperature of 500°F. Stainless steel PH15-7Mo is selected as the alloy to be used for this structure.

Assume:

- o Panel width:  $a = 6$  inches
- o Aspect ratio:  $b/a = 3.0$  ( $F_{11} = 3.33$ )
- o Skin thickness:  $h = 0.050$  inch
- o Damping ratio:  $\zeta = 0.016$
- o Ambient temperature:  $80^\circ\text{F}$

A fatigue curve for the selected alloy at the design temperature was obtained from MIL-HDBK-5B<sup>6</sup> to give the effects of mean stress on fatigue life. (The compressive mean stresses were extrapolated.) This axial loading, constant amplitude, fatigue curve was converted to an equivalent random amplitude fatigue curve (Figure 7) using the method of Reference 7.

From MIL-HDBK-5B, the alloy properties are

$$\begin{aligned}\gamma &= (.277/386) = 7.17 \times 10^{-4} \text{ lb-sec}^2/\text{in}^4 \\ c &= 6.1 \times 10^{-6} \text{ in/in/}^\circ\text{F @ } 500^\circ\text{F} \\ E_o &= 29.0 \times 10^6 \text{ psi @ } 80^\circ\text{F} \\ E &= 26.97 \times 10^6 \text{ psi @ } 500^\circ\text{F}\end{aligned}$$

The skin buckling temperature is calculated from Equation (8):

$$T_c = \frac{5.25(.050)^2(3.33)}{(6.1 \times 10^{-6})(6)(18)(1.32)} = 50.3^\circ\text{F above ambient}$$

The temperature ratio is then

$$r = 420/50.3 = 8.35$$

from Equation (9). The aspect ratio parameter  $R$  is calculated from Equation (11) as

$$R = 3[4.8976(3.33)^2 - 2(5.32)(.68)] = 141.2$$

The buckling amplitude can then be computed from Equation (10):

$$w_o = 2.50(.050)(3.33)^{1.75} [7.35/141.2]^{1/2} = 0.235 \text{ inch}$$

This is the maximum displacement, at the center of the bay, caused by the  $500^\circ\text{F}$  thermal environment. The next step is the computation of the thermal stresses, using equations (14).

REF. 6, FIGURE 2.5.7.1.8(b)  
CONVERTED TO EQUIVALENT RANDOM AMPLITUDE

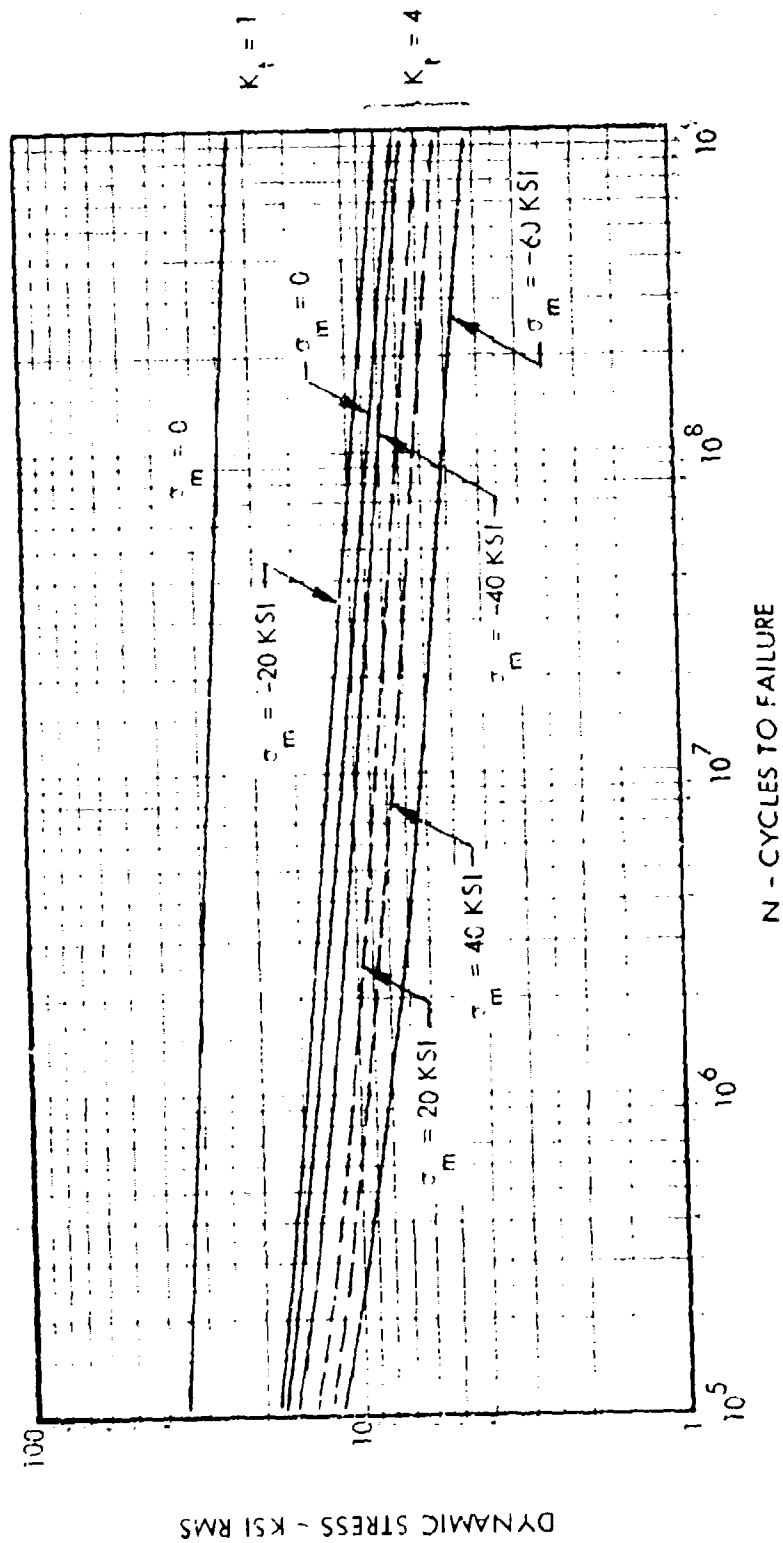


FIGURE 7. RANDOM LOADING FATIGUE CURVE FOR PH15-7Mo STAINLESS STEEL AT 5000F

$$\sigma_T = - \frac{(26.97 \times 10^6)(6.1 \times 10^{-6})(420)}{0.68} = -101.5 \text{ ksi}$$

$$\sigma_{x_b} = \left[ \frac{(0.82)(26.97 \times 10^6)(.235)^2}{(6)(18)(.8976)} \left[ (3.0)(1.8976) + \frac{.32}{3.0} \right] \right] 10^{-3} = 73.1 \text{ ksi}$$

$$\sigma_{y_b} = \left[ \frac{(1.66)(26.97 \times 10^6)(.235)^2}{(6)(18)(.8976)} \left[ \frac{1.8976}{3.0} + (.32)(3.0) \right] \right] 10^{-3} = 40.6 \text{ ksi}$$

and then

$$\sigma_x = -101.5 + 73.1 = -28.4 \text{ ksi}$$

$$\sigma_y = -101.5 + 40.6 = -60.9 \text{ ksi}$$

The ambient temperature fundamental frequency is found from Equation (3) as

$$f_o = \frac{(0.79)(3.33)(.050)}{(6)(18)} \left[ \frac{29.0 \times 10^6}{(7.17 \times 10^{-4})(.8976)} \right]^{1/2} = 259 \text{ Hz}$$

Then the frequency response at a temperature of 500°F is computed from Equation (15) as

$$f(r) = f_o [0.60 + 0.44(7.35)^{1/2}] = 1.79 f_o = 464 \text{ Hz}$$

The acoustic pressure density corresponding to a spectrum level of 140 dB is

$$\ddot{z}(f) = 2.91 \times 10^{[(140/20)-9]} = 2.9 \times 10^{-2} \text{ psi}/\sqrt{\text{Hz}}$$

using Equation (9). From Equation (4) the aspect ratio parameter AR is

$$AR = 3(3.0)^2 + 3/(3.0)^2 + 2 = 29.33$$

This then allows the computation of the dynamic stresses using equations (16) and (17):

$$\tilde{\sigma}_x = 3.6 \times 10^{-4} \left[ \frac{18}{.050} \right]^2 \left[ \frac{2.9 \times 10^{-2}}{29.33} \right] \left[ \frac{464}{.016} \right]^{1/2} = 7.86 \text{ ksi}_{rms}$$

$$\tilde{\sigma}_y = 13.0 \times 10^{-4} \left[ \frac{6}{.050} \right]^2 \left[ \frac{2.9 \times 10^{-2}}{29.33} \right] \left[ \frac{464}{.016} \right]^{1/2} = 3.15 \text{ ksi}_{rms}$$

Enter the fatigue curve of Figure 7 ( $K_f = 4$ ) with the dynamic stress  $\tilde{\sigma}_x = 7.86 \text{ ksi}_{rms}$  and thermal mean stress  $\sigma_x = -28.4 \text{ ksi}$ . This combination gives a life of approximately

$4.5 \times 10^8$  cycles. Using the y-direction stresses,  $\tilde{\sigma}_y = 3.15 \text{ ksi}_{\text{rms}}$  and  $\sigma_y = -60.9 \text{ ksi}$ , gives a life greater than  $10^{10}$  cycles.

At a frequency of 464 Hz, the life is then determined by using the minimum cyclic life,

$$\text{Life} = \frac{N}{3600 f(r)} = \frac{4.5 \times 10^8}{(3600)(464)} = 269 \text{ Hours}$$

which is greater than the 100 hour design requirement. The design is optimized by iteration on the above procedure to decrease the skin gage or increase the stiffener spacing such that the predicted life is equal to or greater than 100 hours.



## IV - DESIGN NOMOGRAPHS

A useful tool for the design engineer is the design nomograph, which graphically displays an equation for rapid solution. The empirical relations of the preceding section were formulated into such nomographs and are presented in this section.

### A. Ambient Temperature Design Criteria

#### 1. Skin Design

Design criteria for stiffened-skin aluminum and titanium structures are displayed in Figure 8 (References 1 and 2). This nomograph is applicable to 7075-T6 aluminum and 6Al-4V titanium.

EXAMPLE: A flat aluminum alloy stiffened structure is to be designed to withstand an estimated sound pressure spectrum level of 120 dB for  $5 \times 10^8$  cycles. The skin design is determined by the procedure described below.

- Assume:
- o Damping ratio:  $\zeta = 0.012$
  - o Panel width:  $a = 4.75$  inches
  - o Aspect ratio:  $b/a = 1.5$

Enter the nomograph, Figure 8, with the design life, and follow through the nomograph, as indicated by the arrows, to obtain a skin thickness  $h = 0.032$  inch.

The fundamental frequency is estimated from Figure 9 (described in the following subsection) as  $f_0 = 340$  Hz. At this frequency, the service environment spectrum level is checked with the spectrum level used above. If necessary, an iteration is made to obtain agreement.

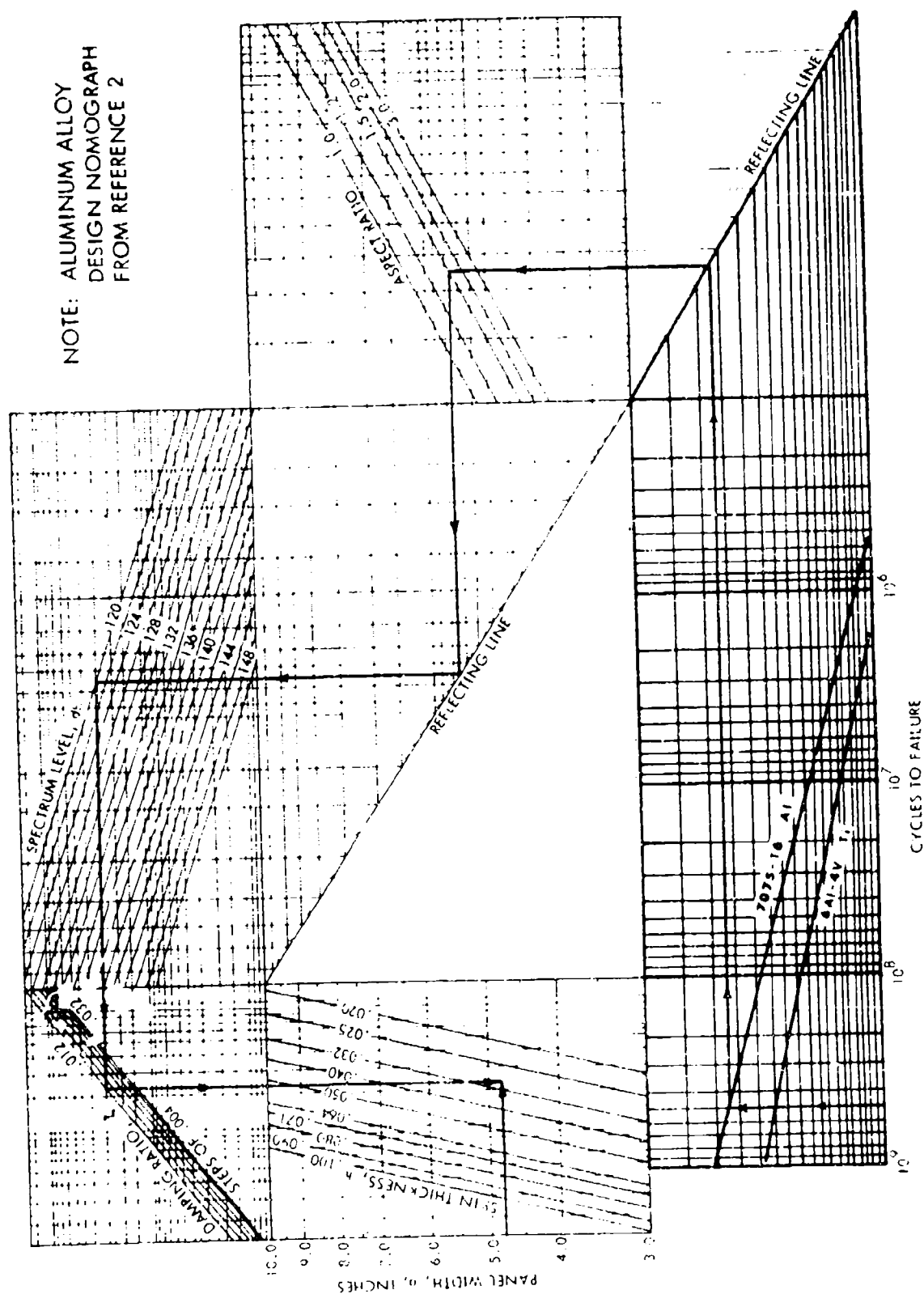


FIGURE 8: STIFFENED PANEL SKIN DESIGN NOMOGRAPH  
AMBIENT TEMPERATURE

## 2. Fundamental Mode Frequency

The fundamental mode frequency for a single bay of a multi-bay structure is given by the nomograph of Figure 9. This nomograph was developed for a constant value of Poisson's ratio,  $\nu = 0.32$ . The chart was simplified by taking advantage of the essentially constant ratio of  $E/\gamma$  for most aircraft structural alloys. An average value of  $E/\gamma = 3.98 \times 10^{10}$  in<sup>2</sup>/sec<sup>2</sup> was used; this is an average of the ratios for aluminum, titanium, stainless steel, and Inconel alloys.

EXAMPLE: The frequency of the structure used for the preceding example is desired.

Panel width:  $a = 4.75$  inches

Aspect ratio:  $b/a = 1.5$

Skin thickness:  $h = 0.032$  inch

Follow the arrows through the nomograph to obtain

$$f_o = 340 \text{ Hz}$$

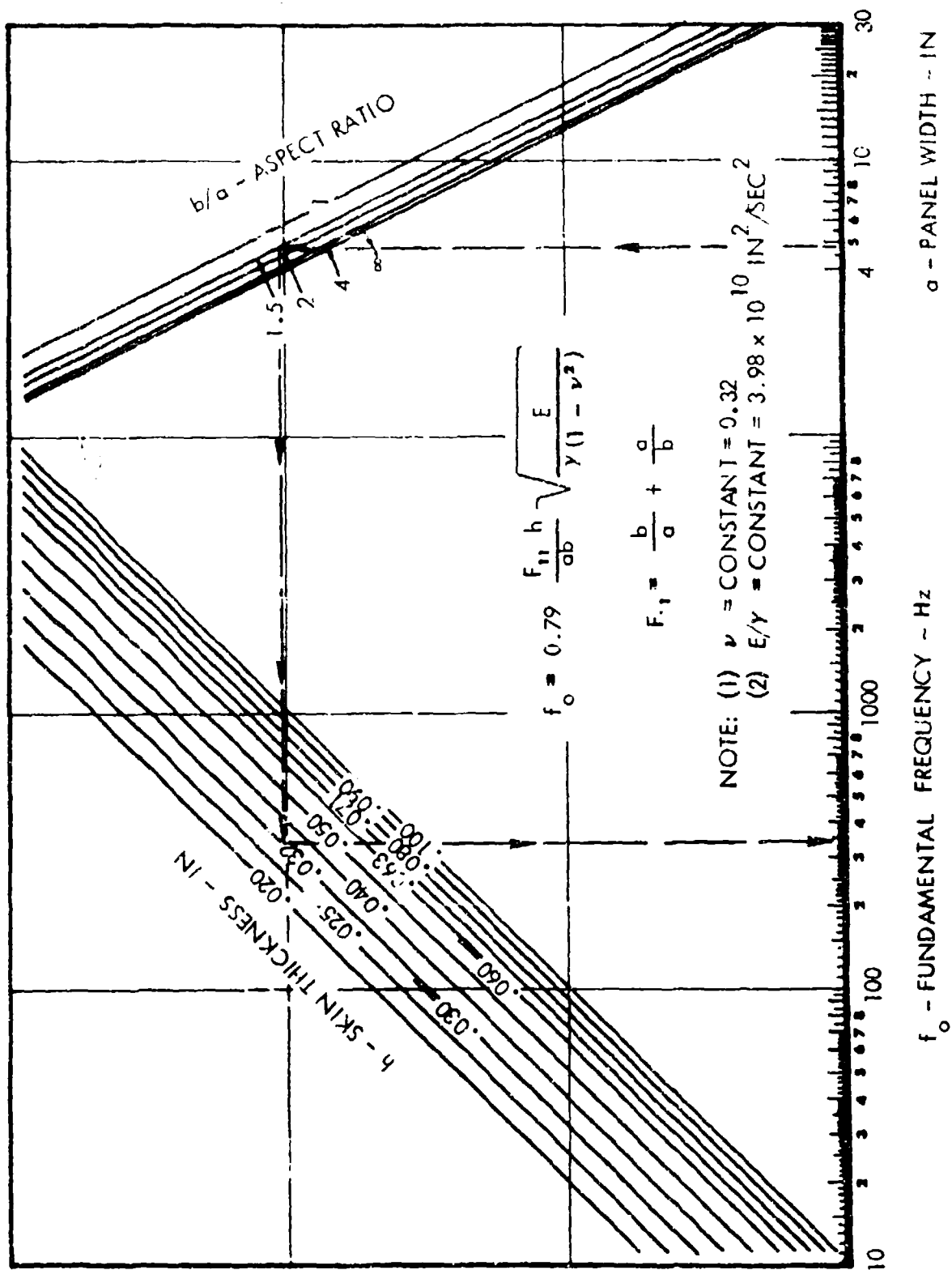


FIGURE 9. AMBIENT TEMPERATURE FUNDAMENTAL FREQUENCY NOMOGRAPH

## B. Elevated Temperature Design Criteria

The examples of nomograph use shown in this subsection are from the example given in Section III.B.

### 1. Skin Buckling Temperature

The critical buckling temperature of a single bay of a multi-bay structure is given by the nomograph of Figure 10. This nomograph is based on a constant value of Poisson's ratio,  $\nu = 0.32$ .

The temperature ratio is then defined as

$$r = T/T_c$$

where both the skin temperature,  $T$ , and the critical buckling temperature,  $T_c$ , are measured relative to the ambient temperature.

EXAMPLE: The skin critical buckling temperature is desired for the structure of the example in Section III.B.

Panel width:  $a = 6$  inches

Aspect Ratio:  $b/a = 3.0$

Skin thickness:  $h = 0.050$  inch

Coefficient of thermal expansion:  $\alpha = 6.1 \times 10^{-6}$  in/in/°F  
(PH15-7Mo Stainless Steel)

Follow the arrows through the nomograph to obtain

$$T_c = 50^\circ\text{F above ambient,}$$

then the temperature ratio is

$$r = 420/50 = 8.4$$

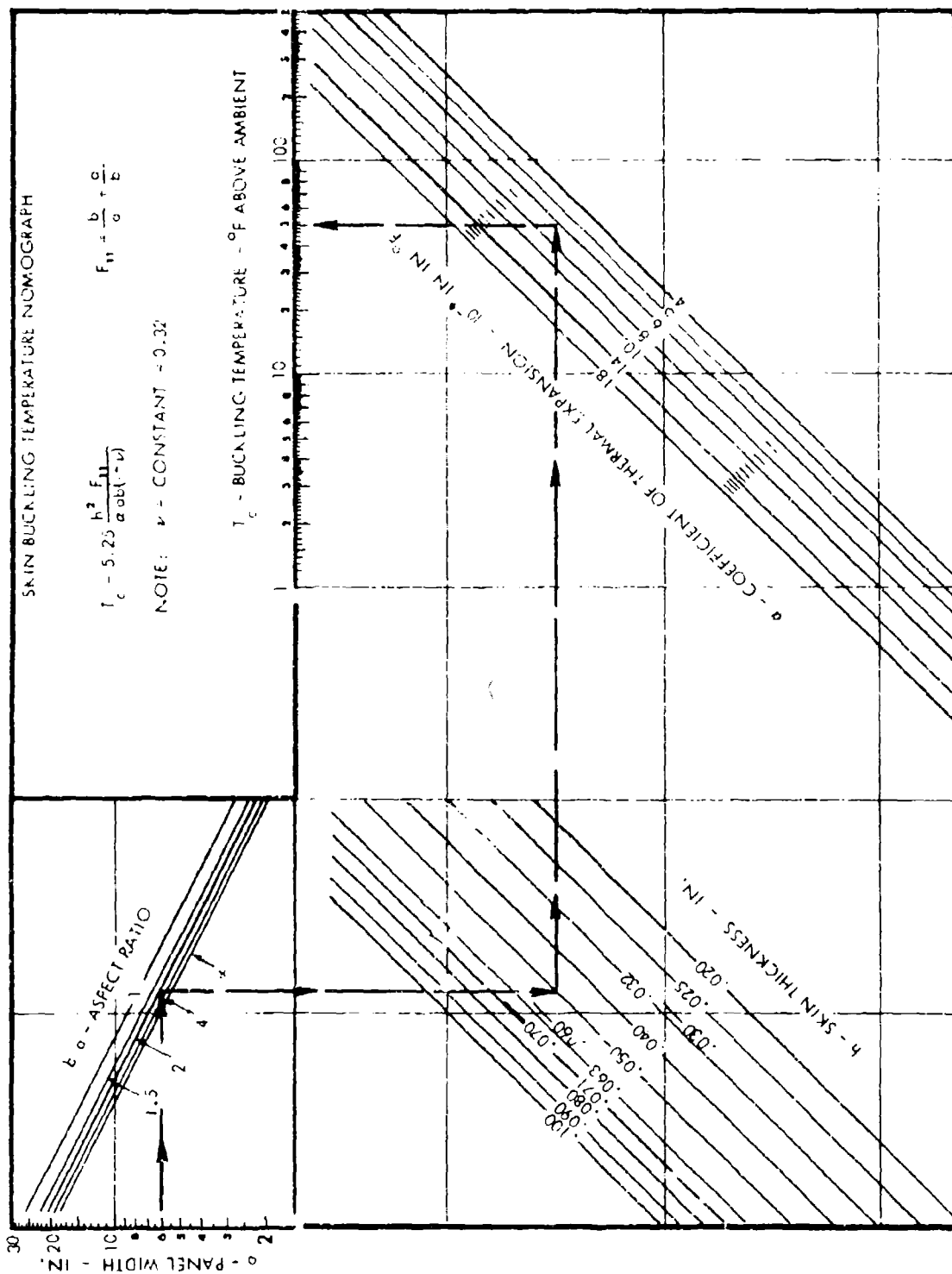


FIGURE 10. SKIN BUCKLING TEMPERATURE NOMOGRAPH

## 2. Skin Buckling Amplitude

The skin buckling amplitude is given by the nomograph of Figure 11. This is the maximum buckle amplitude on a single bay of a multi-bay structure; it occurs at the midpoint of the bay. The nomograph was developed for a constant value of Poisson's ratio,  $\nu = 0.32$ .

The buckling amplitude is dependent on the temperature ratio as given by the nomograph of Figure 10.

**EXAMPLE:** The skin buckling amplitude is desired for the structure of the example in Section III.B.

Skin temperature:  $T = 420^\circ\text{F}$  above ambient

Panel width:  $a = 6$  inches

Aspect ratio:  $b/a = 3.0$

Skin thickness:  $h = 0.050$  inch

From the preceding example, the temperature ratio is  $r = 8.4$ . Enter the nomograph and follow through the arrows to obtain the buckling amplitude

$$W_o = 0.24 \text{ inch}$$

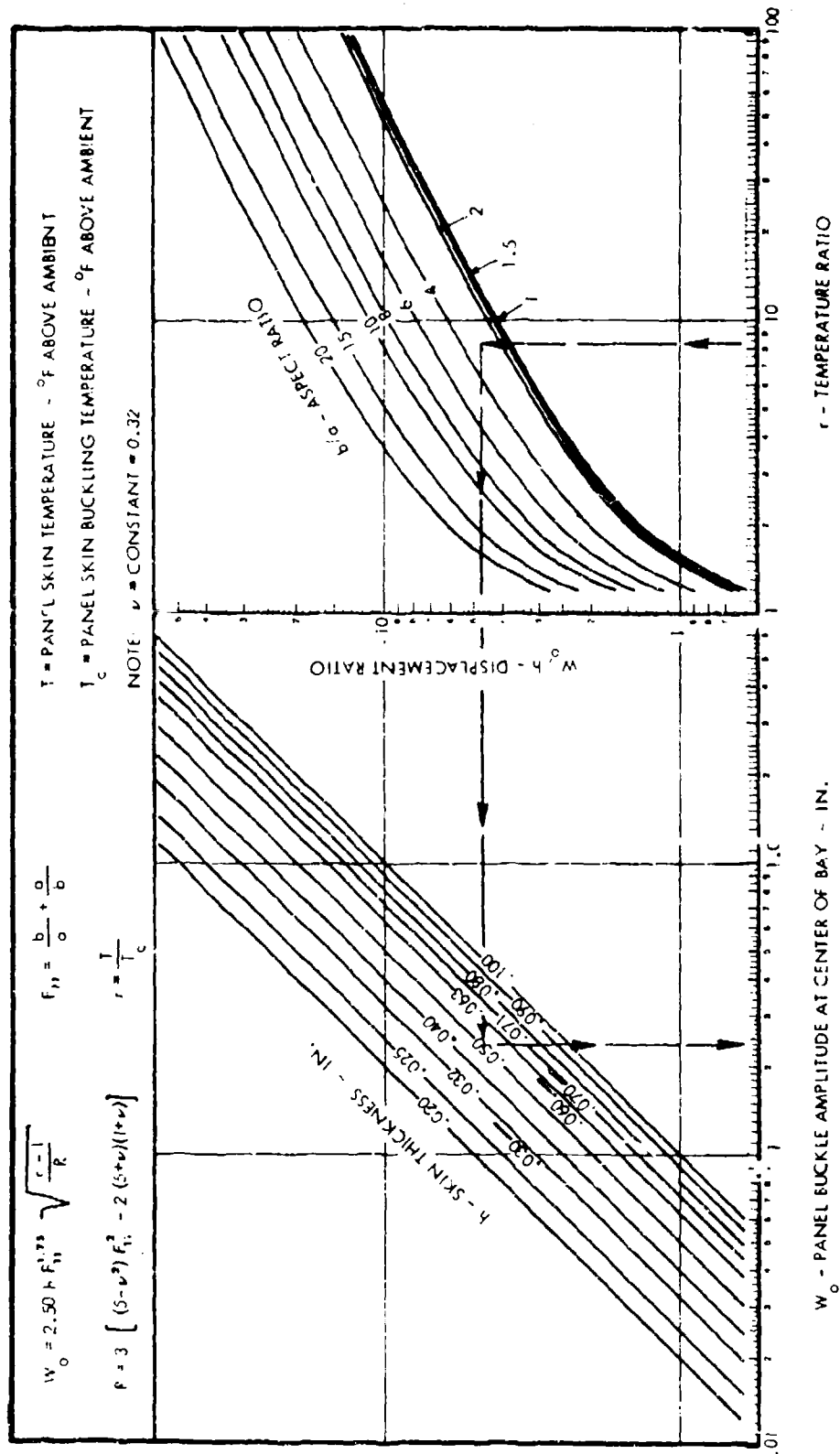


FIGURE 11. SKIN BUCKLING AMPLITUDE NOMOGRAPH



### 3. Thermal Stress

Thermal stresses due to in-plane expansion and skin buckling are given by Figures 12 through 14. The thermal stresser are then the sum of these component stresses in the applicable direction:

- o Midpoint of panel long side, in x -direction

$$\sigma_x = \sigma_T + \sigma_{x_b}$$

- o Midpoint of panel short side, in y -direction

$$\sigma_y = \sigma_T + \sigma_{y_b}$$

The in-plane expansion stress is compressive (-), while the buckling stresses  $\sigma_{x_b}$  and  $\sigma_{y_b}$  are tensile (+); the sign must be maintained throughout the computation.

The value of Poisson's ratio has been held constant at  $\nu = 0.32$  for development of these nomographs.

The skin buckling amplitude from Figure 11 is required for solution of the thermal stresses.

**EXAMPLE:** The thermal stresses are desired for the structure of the example in Section III.B.

Skin temperature:  $T = 420^\circ\text{F}$  above ambient

Panel width:  $a = 6$  inches

Aspect ratio:  $b/a = 3.0$

Coefficient of thermal expansion:  $\alpha = 6.1 \times 10^{-6}$  in./in./ $^\circ\text{F}$  @  $500^\circ\text{F}$

Modulus of Elasticity:  $E = 26.97 \times 10^6$  psi @  $500^\circ\text{F}$

Enter Figure 12 with the required parameters, and follow the arrows to obtain

$$\sigma_T = -100 \text{ ksi}$$

From the preceding example, the skin buckle amplitude is 0.24 inch. Enter Figures 13 and 14 with the buckle amplitude  $W_0 = 0.24$ , and follow the arrows through the nomograph to obtain

$$\sigma_{x_b} = 75 \text{ ksi}; \quad \sigma_{y_b} = 40 \text{ ksi}$$

Then

$$\sigma_x = -100 + 75 = -25 \text{ ksi}$$

$$\sigma_y = -100 + 40 = -60 \text{ ksi}$$

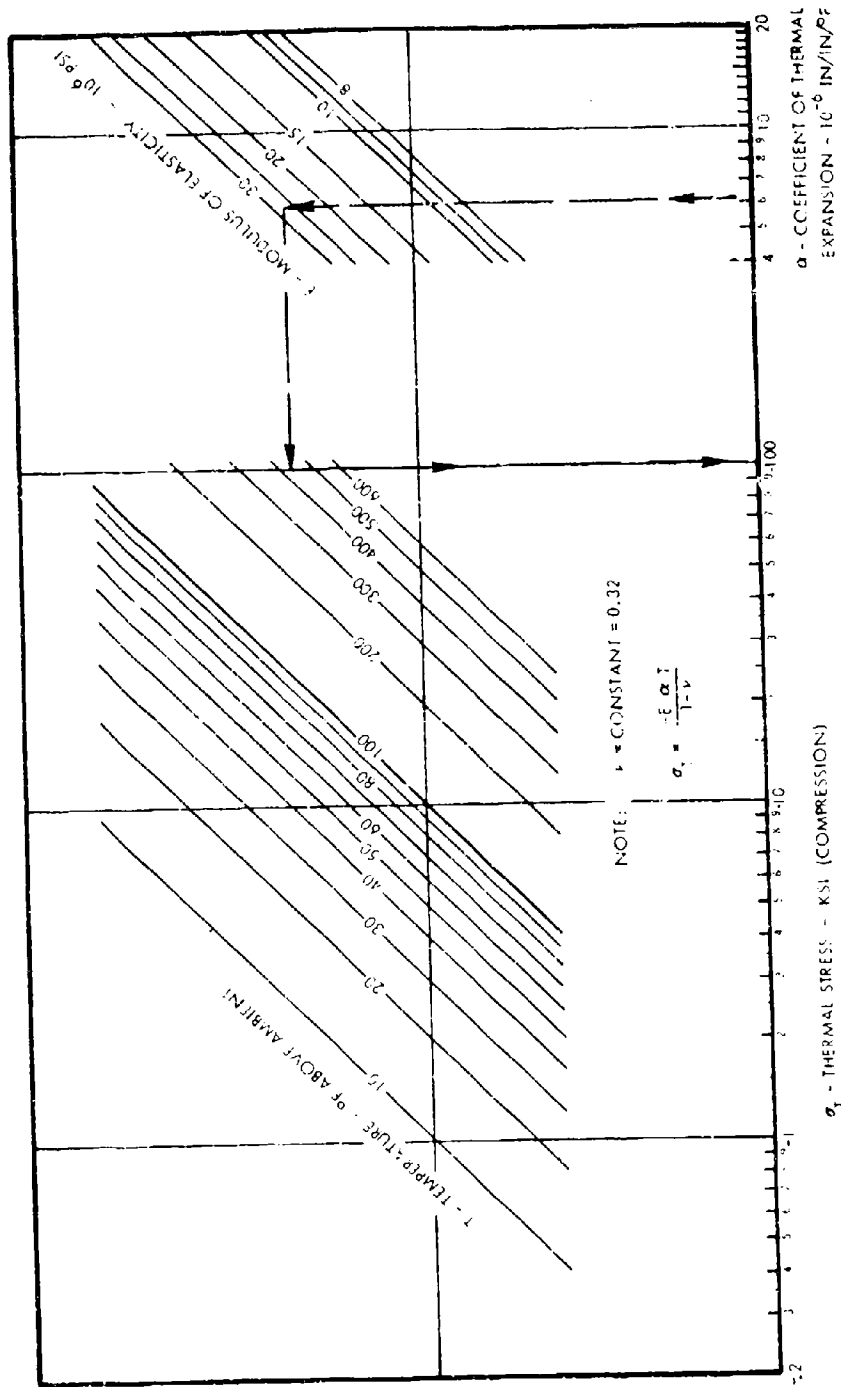


FIGURE 12. THERMAL EXPANSION STRESS NOMOGRAPH

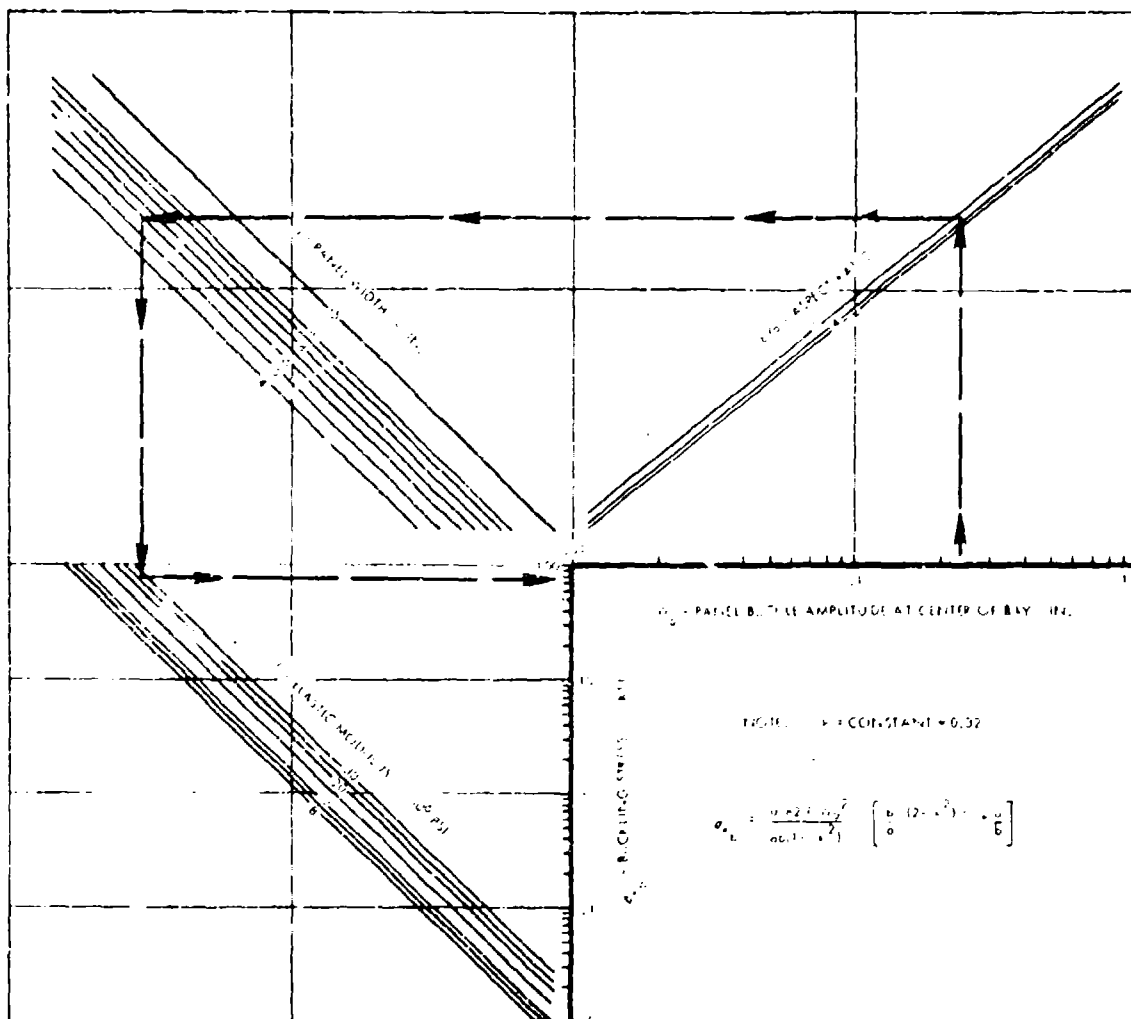


FIGURE 13. THERMAL BUCKLING STRESS NOMOGRAPH  
X - DIRECTION STRESS

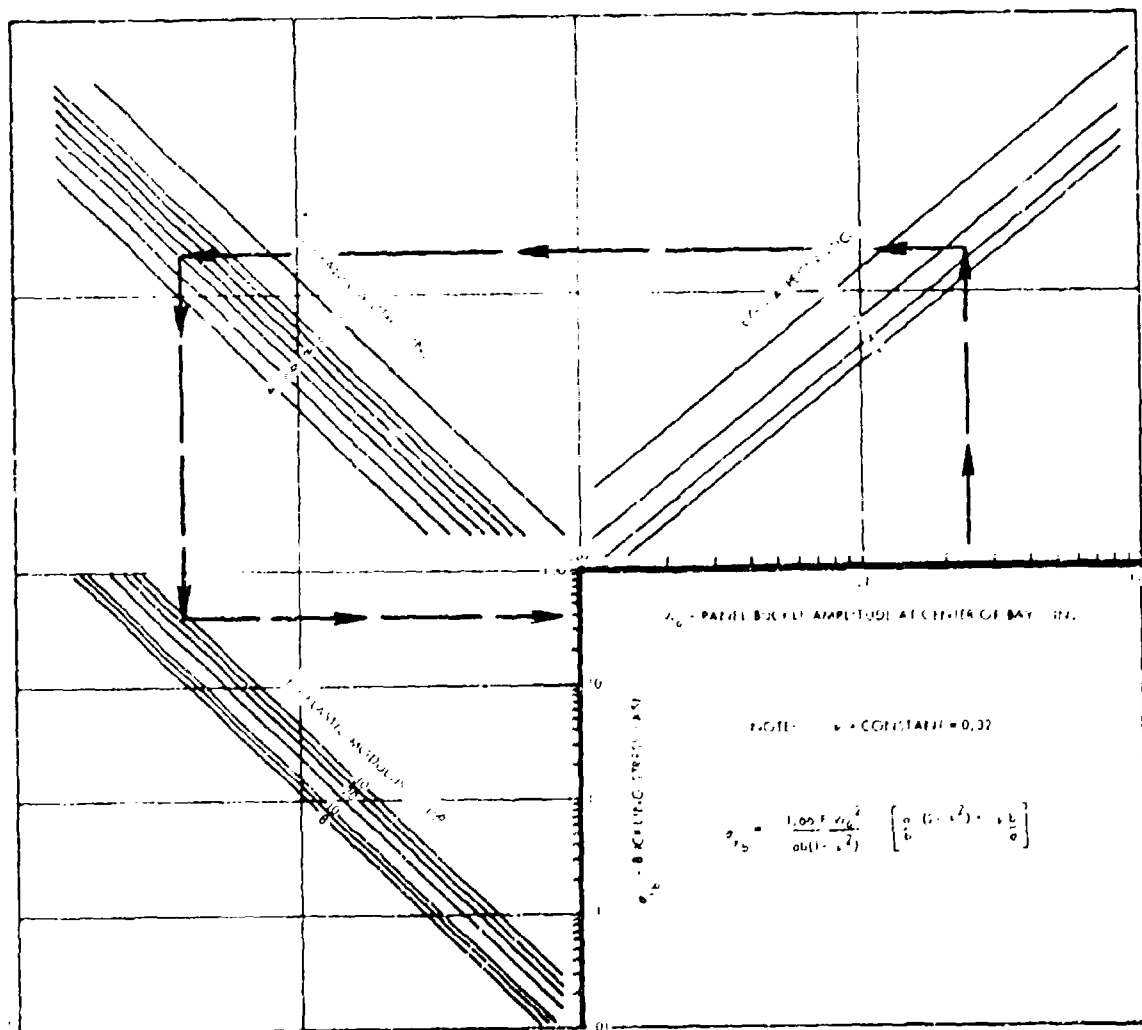


FIGURE 14. THERMAL BUCKLING STRESS NOMOGRAPH  
Y - DIRECTION STRESS

#### 4. Elevated Temperature Frequency Response

The fundamental mode frequency decreases with increasing temperature until the skin buckles; the frequency then increases with further increases in temperature. The nomograph of Figure 15 gives the elevated temperature frequency response as a function of the ambient temperature frequency.

**EXAMPLE:** The elevated temperature frequency response is desired for the structure in the example of Section III.B.

Skin temperature:  $T = 420^\circ\text{F}$  above ambient

Panel width:  $a = 6$  inches

Aspect ratio:  $b/a = 3.0$

Skin thickness:  $h = 0.050$  inch

From the preceding examples in this Section, the temperature ratio is  $r = 8.4$ . From Figure 9, the fundamental frequency for this configuration is found to be

$$f_o = 260 \text{ Hz}$$

From the nomograph of Figure 15, the frequency ratio is

$$f(r)/f_o = 1.8$$

from which the frequency response at the temperature,  $T$ , is

$$f(r) = 1.8 f_o = 1.8 (260) = 468 \text{ Hz}$$

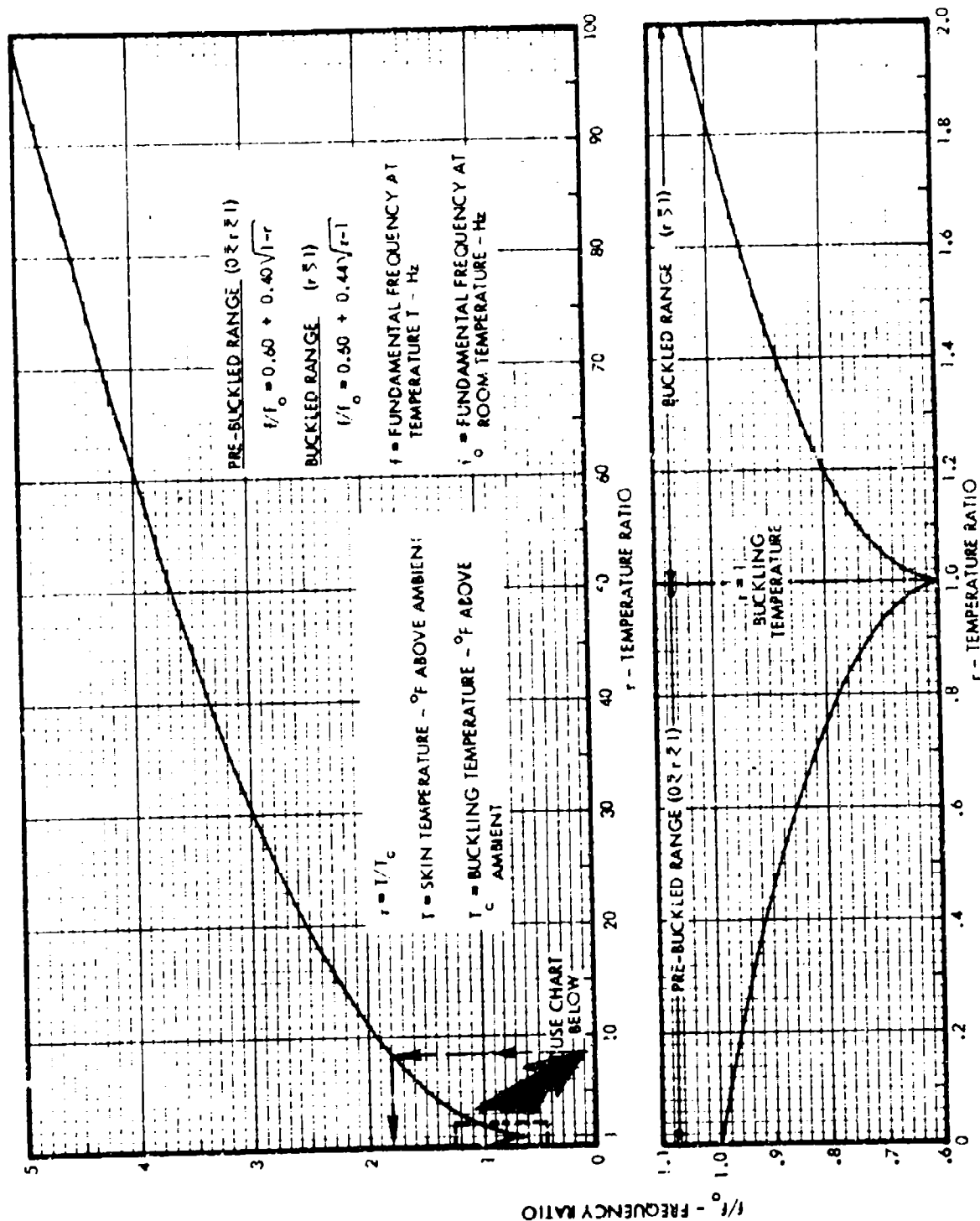


FIGURE 15. ELEVATED TEMPERATURE FUNDAMENTAL FREQUENCY NOMOGRAPH

## 5. Fatigue Life

The nomograph of Figure 16 is used for elevated temperature applications for 7075-T6 aluminum at temperatures of 150° and 300°F and 6Al-4V titanium at temperatures of 400° and 600°F. This nomograph includes thermal mean stress effects in the data, thereby negating the need to compute these parameters.

EXAMPLE: A flat aluminum (7075-T6) structure is to be designed for a service life of 100 hours at a sound pressure spectrum level of 120 dB and a service temperature of 300°F.

- Assume:
- o Aspect ratio:  $b/a = 3.0$  ( $F_{11} = 3.33$ )
  - o Damping ratio:  $\zeta = 0.016$
  - o Ambient temperature: 80°F

As a first step, assume a frequency, at the service temperature, of 300 Hz. Then the life in cycles is

$$N = (300 \text{ Hz})(100 \text{ Hrs})(3600 \text{ sec/Hr}) = 1.08 \times 10^8 \text{ cycles}$$

Enter the nomograph of Figure 16 with this life and follow through the parameters to the skin thickness chart. Several spacing/skin thickness ratios are now possible, all of which will meet the design life goal. Assuming a skin thickness of  $h = 0.050$  inch, the panel width is found to be  $a = 5.0$  inches.

At this point a structural configuration is defined, however, the assumed frequency must be checked. Compute the fundamental mode frequency at the ambient temperature using Figure 9, ( $f_0 = 390$  Hz) then compute the skin critical buckling temperature using Figure 10, where  $\alpha = 13.5 \times 10^{-6}$  in/in/°F from Figure 18; ( $T_c = 33^\circ\text{F}$ ). The temperature ratio is then

$$r = T/T_c = 220/33 = 6.7$$

Figure 15 then yields the frequency ratio  $f(r)/f_0 = 1.65$ , and the elevated temperature frequency is

$$f(r) = 1.65 f_0 = 1.65(390) = 645 \text{ Hz}$$

Since this frequency is greater than the assumed frequency, the anticipated life will be less than the design goal, hence an iteration is necessary. The above procedure is repeated using the calculated frequency of 645 Hz at the design temperature. One or more iterations may be necessary to obtain agreement between the initial and final frequency (or design life).

Several spacing/skin thickness ratio combinations may be derived using this method and the weight of each calculated to obtain a minimum weight design.

This method may also be used for structural temperatures different than those indicated on the nomograph by assuming a linear relationship between the temperatures shown and interpolating.





## 6. Application of the Design Procedure

At least two alternative methods of application are possible using the elevated temperature design criteria. The first of these, and the most direct, involves the use of the life design nomograph, Figure 16. The alloys and structural temperatures must match those for which the nomograph was developed. An example of the use of this nomograph was presented in the preceding subsection.

a. Mean Stress Fatigue Curve - An alternate design method involves the use of fatigue curves where the mean stress effects are known (see Figure 7). This method can be employed where the alloy or temperature does not coincide with those of the Figure 16 nomograph. The following example is identical to the example of Section III.B and to the examples used on each of the design nomographs of this section. It is presented here to summarize the application of the design chart section.

EXAMPLE: A flat stiffened structure is to be designed for a service life of 100 hours at a sound pressure spectrum level of 140 dB and a service temperature of 500 °F. Stainless steel PH15-7Mo is selected as the alloy to be used for this structure.

- Assume:
- o Panel width:  $a = 6$  inches
  - o Aspect ratio:  $b/a = 2.0$  ( $F_{11} = 3.33$ )
  - o Skin thickness:  $h = 0.050$  inch
  - o Damping ratio:  $\zeta = 0.016$
  - o Ambient temperature: 80°F

Figure 7 shows a random amplitude fatigue curve with varying mean stresses for the PH15-7Mo alloy.

From MIL-HDBK-5B, the alloy properties are

$$\gamma = (.277/386) = 7.17 \times 10^{-4} \text{ lb-sec}^2/\text{in}^4$$

$$\alpha = 6.1 \times 10^{-6} \text{ in/in/}^\circ\text{F @ } 500^\circ\text{F}$$

$$E_o = 29.0 \times 10^6 \text{ psi @ RT}$$

$$E = 26.97 \times 10^6 \text{ psi @ } 500^\circ\text{F}$$

The skin critical buckling temperature is found from Figure 10 as  $T_c = 50^\circ\text{F}$  above ambient. Then  $r = 420/50 = 8.4$  and the buckling amplitude is  $W_o = 0.24$  inch from Figure 11.

The thermal stress components at the midpoints of the two sides are found from Figures 12 through 14, or

$$\sigma_T = -100 \text{ ksi}$$

$$\sigma_{x_b} = 75 \text{ ksi} ; \sigma_{y_b} = 40 \text{ ksi}$$

Then the thermal (compressive) stresses are

$$\sigma_x = \sigma_T + \sigma_{x_b} = -100 + 75 = -25 \text{ ksi}$$

$$\sigma_y = \sigma_T + \sigma_{y_b} = -100 + 40 = -60 \text{ ksi}$$

The ambient temperature fundamental frequency is  $f_o = 260 \text{ Hz}$  from Figure 9. Figure 15 gives a frequency ratio of 1.8, corresponding to a temperature ratio of 8.4; then

$$f(r) = 1.8 f_o = 1.8(260) = 468 \text{ Hz}$$

The dynamic stresses are then computed by Equations (16) and (17):

- o Midpoint of panel long side

$$\tilde{\sigma}_x = 3.60 \times 10^{-4} \left[ \frac{18}{0.050} \right]^2 \frac{2.9 \times 10^{-2}}{29.33} \left[ \frac{468}{0.016} \right]^{1/2} = 7.89 \text{ ksi}_{\text{rms}}$$

- o Midpoint of panel short side

$$\tilde{\sigma}_y = 13.0 \times 10^{-4} \left[ \frac{6}{0.050} \right]^2 \frac{2.9 \times 10^{-2}}{29.33} \left[ \frac{468}{0.016} \right]^{1/2} = 3.17 \text{ ksi}_{\text{rms}}$$

The value of  $\ddot{z}(f) = 2.9 \times 10^{-2}$  is the acoustic pressure density corresponding to 140 dB (Equation (5)), while  $AR = 29.33$  is the aspect ratio parameter (Equation (4b)).

Enter the fatigue curve of Figure 7 ( $K_f = 4$ ) with the dynamic stress  $\tilde{\sigma}_x = 7.89 \text{ ksi}_{\text{rms}}$  and thermal mean stress  $\sigma_x = -25 \text{ ksi}$ . This combination gives a life of approximately  $4.5 \times 10^8$  cycles. Using the y-direction stresses,  $\tilde{\sigma}_y = 3.17 \text{ ksi}_{\text{rms}}$  and  $\sigma_y = -60 \text{ ksi}$ , gives a life greater than  $10^{10}$  cycles. At a frequency of 468 Hz, the life is then determined by using the minimum cyclic life,

$$\text{LIFE} = \frac{N}{3600f} = \frac{4.5 \times 10^8}{(3600)(468)} = 267 \text{ Hours}$$

which is greater than the 100 hour design requirement. The design can be optimized by iteration on the above procedure to decrease the skin gage or increase the stiffener spacing such that the predicted life is equal to or greater than the 100 hour design life. Alternatively, the above design may be used and the additional estimated life used as a safety factor.

A comparison of the results achieved here (using the design charts) and the results of Section III.B (using the design equations) shows a difference of only 2 hours (less than 1% error) on the final life. Hence the accuracy of the graphical method of solution is compatible with that of the computational method.

After optimizing the above design, the stiffener flange details can be determined using the method of Section III.A. The elevated temperature response frequency calculated above (i.e.,  $f(r) = 468$  Hz) should be used for this computation. The fatigue data used in the skin design (i.e., Figure 7 for this example) may be used for the stiffener design in lieu of a fatigue curve developed specifically for stiffeners at the elevated temperature.

## V - COMPUTER PROGRAMS

A digital computer program is presented here for calculating the dynamic response and life of a multi-bay, flat stiffened-skin structure exposed to simultaneous acoustic and thermal environments. Five sub-programs are required for the dynamic analysis program. These programs were developed for the Univac 1106 computer using Fortran V; however, the programs can readily be adapted to any digital computer.

### A. Analysis Program

The input data format for the analysis program is shown in Table I and the input parameters are defined in Table II. The computer program is listed in Table III while Table IV contains a sample of the output format.

### B. Sub-programs Required

The following sub-programs are required for this analysis program:

ETEMP (T, IFF)  
ALPHA (T, IFF)  
SN (SDYN, STEMP, TEMP, CTF, IFF)  
CTEMP (TCALP, TC, IFF)  
PROP (OPT, B, H, T, A, RJ, GAMAT, XIP)

The input and output parameters for each sub-program are given in the following sub-sections. A listing of each of the sub-programs is contained in Tables V through IX. Since each of these sub-programs are either functions or subroutines, the input and output are controlled by the calling program.

#### 1. Sub-programs ETEMP and ALPHA

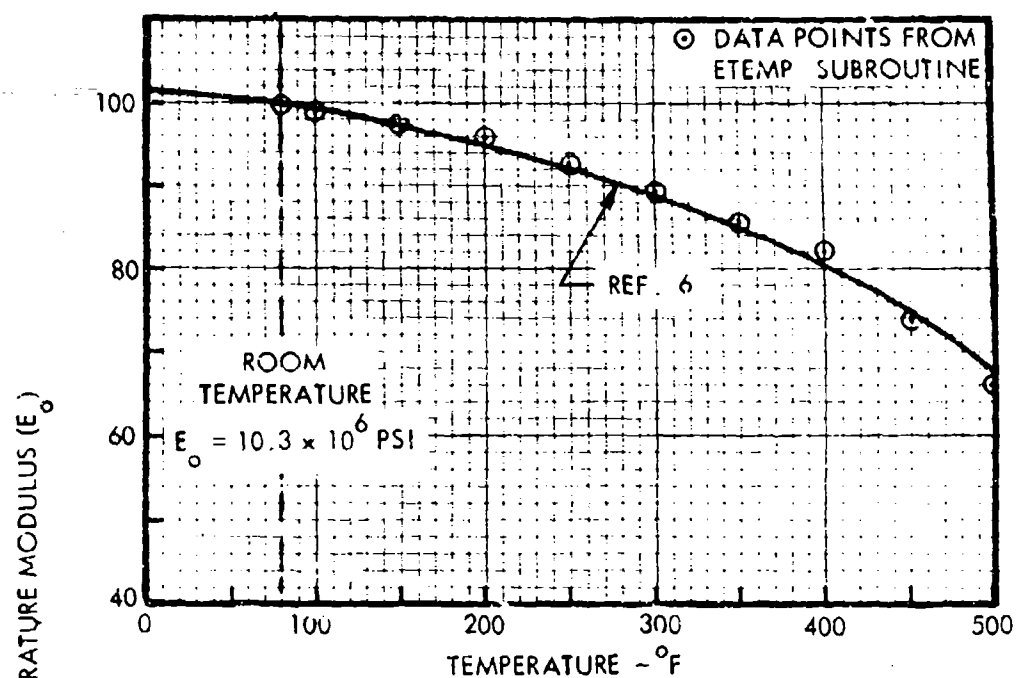
These functions compute modulus of elasticity and coefficient of thermal expansion, respectively, for aluminum and titanium alloys as a function of temperature. The basic alloy properties are from MIL-HDBK-5B, and are shown in Figures 17 and 18. The input parameters are:

T - Input Temperature at which elastic modulus or coefficient of thermal expansion desired - °F

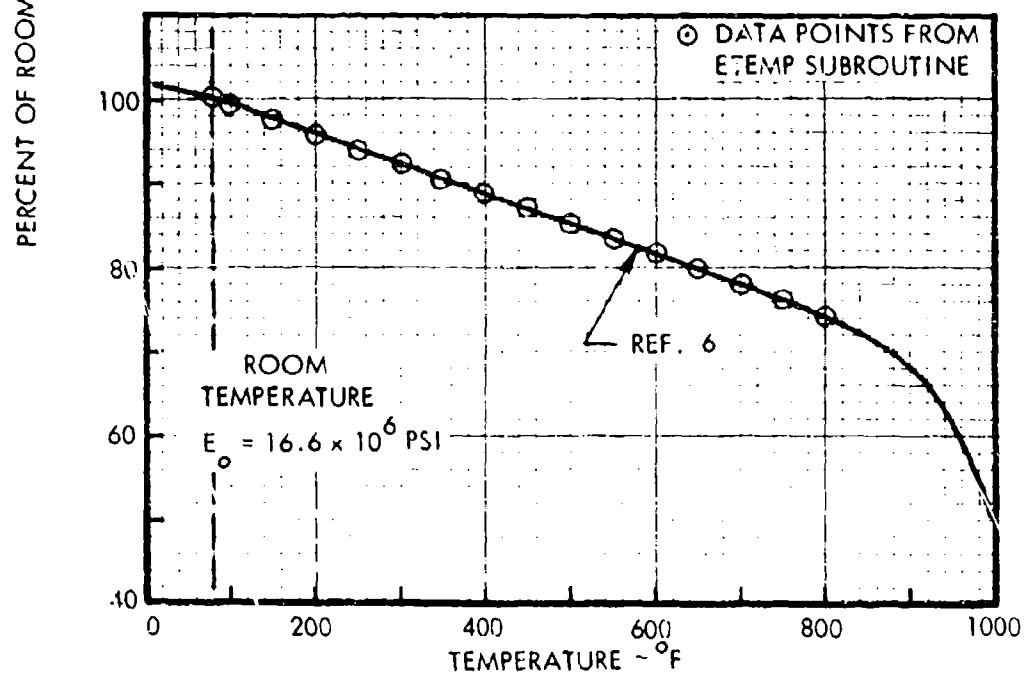
IFF - Alloy code,

= 1 Titanium Alloy (6Al-4V annealed)

= 2 Aluminum Alloy (7075-T6)

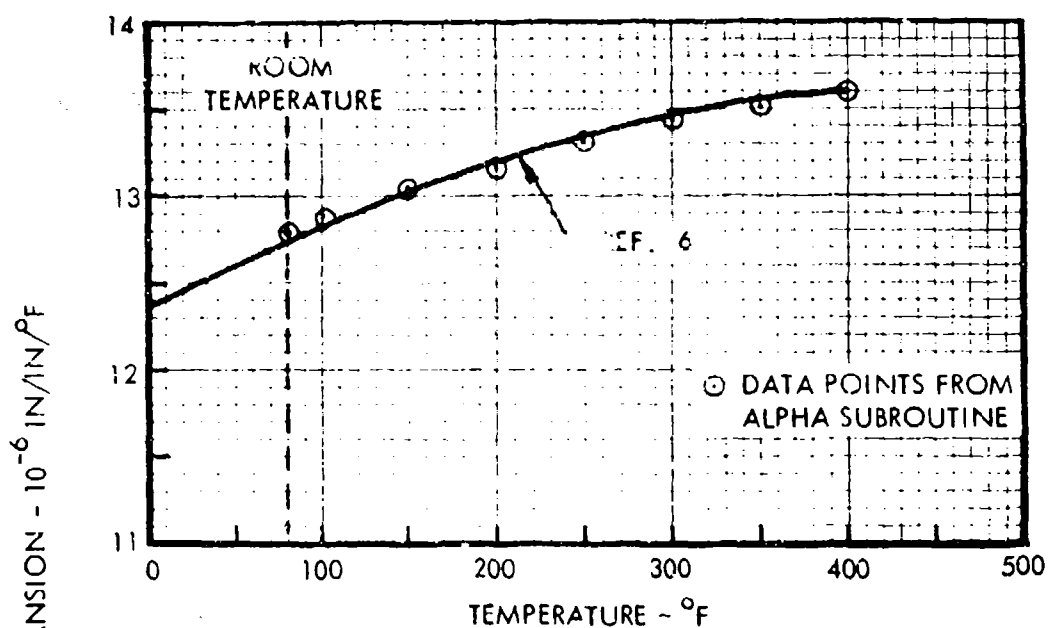


a) ALUMINUM ALLOY 7075-T6 SHEET

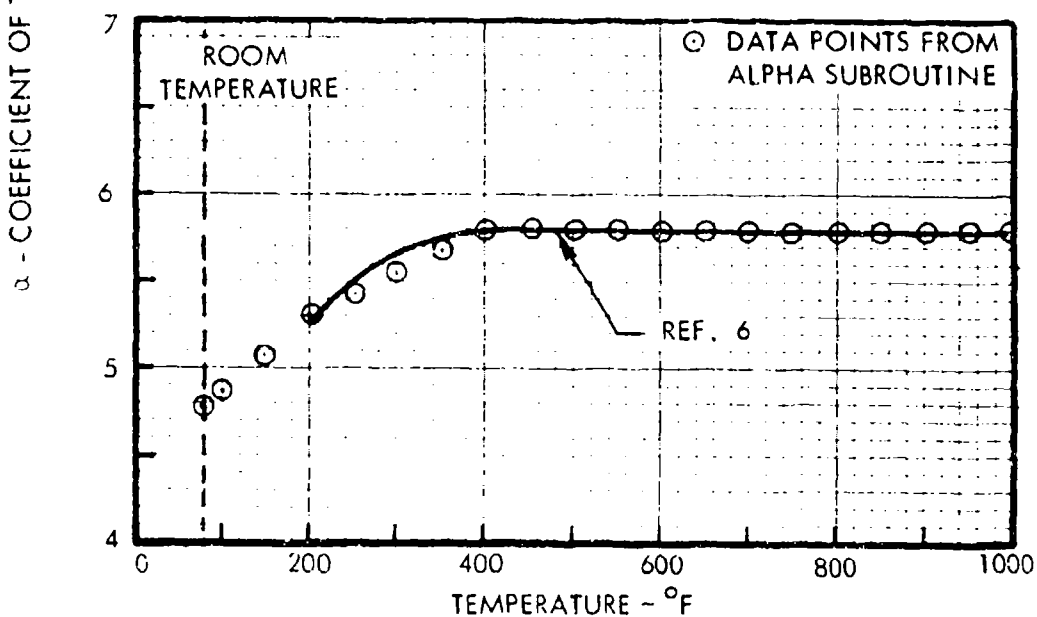


b) TITANIUM ALLOY 6Al-4V ANNEALED SHEET

FIGURE 17. TEMPERATURE EFFECTS ON ELASTIC MODULUS  
(FROM REFERENCE 6)



a) ALUMINUM ALLOY 7075-T6



b) TITANIUM ALLOY 6Al-4V

FIGURE 18. TEMPERATURE EFFECTS ON COEFFICIENT OF THERMAL EXPANSION (FROM REFERENCE 6)

## 2. Sub-program SN

This subroutine computes the fatigue life of aluminum and titanium alloys as a function of dynamic and thermal mean stresses. The input parameters are:

- SDYN - Dynamic Stress - ksi rms
- STEMP - Thermal (or Mean) Stress - ksi
- TEMP - Temperature - °F
- IFF - Alloy Code,
  - = 1 Titanium alloy (6Al-4V annealed)
  - = 2 Aluminum Alloy (7075-T6)

The output to the calling program is:

- CTF - Life in cycles to failure

## 3. Sub-program CTEMP

This subroutine computes the skin critical buckling temperature for aluminum or titanium structures. The input parameters are:

- TCALP - Product of critical buckling temperature and coefficient of thermal expansion; computed using Equation (8).
- IFF - Alloy code,
  - = 1 Titanium Alloy (6Al-4V Annealed)
  - = 2 Aluminum Alloy (7075-T6)

The single output parameter to the calling program is:

- TC - Critical buckling temperature - °F above ambient.

## 4. Sub-program PROP

This program computes stiffening member properties such as area and moment of inertia. The basic relations are from Reference 3 and are shown in Appendix I. Two different sectional shapes are available, a zee or a channel section, with the parameters described in Appendix I.

The input parameters are

- OPT - Option code to select sectional shape,
  - = 0 zee-section
  - = 1 channel section
- B - Flange width of stiffening member - inch
- H - Height of stiffening member - inch
- T - Thickness of stiffening member - inch

The output parameters to the calling program are:

- A - Cross-sectional area -  $\text{in}^2$
- RJ - St. Venant's Torsion Constant -  $\text{in}^4$
- GAMAT - Warping constant for thin-walled open-section beam, with the pole taken at the shear center -  $\text{in}^6$
- XIP - Polar moment of inertia, referenced to rotation about the attachment point -  $\text{in}^4$



# TABLE I

## DYNAMIC ANALYSIS COMPUTER PROGRAM INPUT FORMAT

### CARD 1

NAME	NCASE	IFF
COL(FORMAT)	1(I2)	3(I2)

### CARD 2

NAME	OPTX	BX	HX	TX
COL(FORMAT)	1(I2)	3(F8.4)	11(F8.4)	19(F8.4)

### CARD 3

NAME	OPTY	BY	HY	TY
COL(FORMAT)	1(I2)	3(F8.4)	11(F8.4)	19(F8.4)

### CARD 4

NAME	A1	A2	B2	B1
COL(FORMAT)	1(F8.4)	9(F8.4)	17(F8.4)	25(F8.4)

### CARD 5

NAME	TS	RHO	RNU	DAMP
COL(FORMAT)	1(F8.4)	9(F8.4)	17(F8.4)	25(F8.4)

### CARD 6

NAME	PSL	T
COL(FORMAT)	1(F8.4)	9(F8.4)

TABLE - II  
DYNAMIC ANALYSIS COMPUTER PROGRAM  
INPUT PARAMETER DEFINITION

NCASE		Two-digit identification number
IFF		Alloy identification code, =1 Titanium Alloy (6A1-4V Annealed) =2 Aluminum Alloy (7075-T6)
OPTX	}	Input parameters defining stiffening member parallel to x-direction - see Sub-program PROP for definition
BX		
HX		
TX		
OPTY	}	Input parameters defining stiffening member parallel to y-direction - see Sub-program PROP for definition
BY		
HY		
TY		
A1	}	Panel bay dimensions - see Figure 2
A2		
B1		
B2		
TS		Skin thickness - inch
RHO		Weight density of skin and stiffening member alloy - lb/in <sup>3</sup>
RNU		Poisson's ratio for structure alloy
DAMP		Damping ratio for structure
PSL		Sound pressure spectrum level - dB
T		Structure temperature rise - °F above ambient

TABLE III  
COMPUTER PROGRAM FOR ELEVATED TEMPERATURE  
DYNAMIC RESPONSE OF STIFFENED STRUCTURE

```

1      C      THIS PROGRAM CALCULATES THE DYNAMIC RESPONSE OF
2      C      A NINE-BAY FLAT STIFFENED PANEL EXPOSED TO A
3      C      UNIFORM ACOUSTIC PRESSURE AND A UNIFORM TEMP-
4      C      ERATURE RISE. ROOM TEMPERATURE IS 80 DEGREES F.
5      C
6      C      T IS A TEMPERATURE RISE, ABOVE ROOM TEMPERATURE
7      C
8      C      SUBPROGRAMS REQUIRED: ALPHA(T,IFF), ETEMP(T,IFF),
9      C      SN(SDYN,STEMP,T,CTF,IFF), CTEMP(TCALP,TC,IFF),
10     C      AND PROP(OPT,B,H,T,A,ZJ,WC,PIP)
11     C
12     C      FUNCTION DEFINITION
13     C
14     F(B,A)=B/A+A/B
15     R(B,A,PR)=3.*((5.-PR+.2)*(B/A+A/B)*.2-.2*(5.+PR)
16     C      *(1.-PR))
17     200 READ(5,301)NCASE,IFI
18     READ(5,302)OPTX,BX,HX,TX,AX,XJ,WCX,XI
19     READ(5,302)OPTY,BY,HY,TY,AY,YJ,WCY,YI
20     READ(5,301)KX
21     READ(5,303)A1,A2,B2,B1
22     READ(5,303)TS,RHO,RNU,DAMP
23     READ(5,303)PSL,T
24     C      INPUT DATA FORMAT STATEMENTS
25     301 FORMAT(2I2)
26     302 FORMAT(12,3F8.4)
27     303 FORMAT(4F8.4)
28     C      CALCULATE SUBSTRUCTURE PROPERTIES
29     CALL PROP(OPTX,BX,HX,TX,AX,XJ,WCX,XI)
30     CALL PROP(OPTY,BY,HY,TY,AY,YJ,WCY,YI)
31     H=TS
32     GM=RHO
33     PR=RNU
34     C      CALCULATE STIFFENED STIFFNES, AND MAG
35     RX1=0.0506 *A1+A1*XJ/(WCX*(1.+PR))
36     RX2=0.0506 *A2+A2*XJ/(WCX*(1.+PR))
37     RY1=0.0506 *B1+B1*YJ/(WCY*(1.+PR))
38     RY2=0.0506 *B2+B2*YJ/(WCY*(1.+PR))
39     SKX=WCX*(1.+RX2+.2*(A2/A1)+(1.+RX1)/A2
40     SKY=WCY*(1.+RY2+.2*(B2/B1)+(1.+RY1)/B2
41     H3=H+H1
42     STR=473.7408*(1.-PR+PR)+(SKX+SKY)/(H3+A2+B2)
43     STR=STR/40
44     A3=(A1/A2)*.3
45     B3=(B1/B2)*.3

```

# TABLE III (CONT'D)

```

46      GM=6./386.
47      SKM=0.25*GM*(1./A2+1./B2)*(1.+2.*A3+2.*B3+4.*A3*B3)
48      1      +9.8696*GM*(X1+A2)*(1.+2.*A3)/(B2*B2)
49      2      +Y1+B2*(1.+2.*B3)/(A2*A2)
50      C      CALCULATE COVER SHEET STIFFNESSES AND MAS
51      F2=F(B2,A2)
52      F21=F(B2,A1)
53      F12=F(B1,A2)
54      F1=F(B1,A1)
55      F15=F2 *F2 +1.*(A1/A2)*F21+F21
56      1      +2.*(B1/B2)*F12+F12+4.*(A1/A2)*(B1/B2)*F1 *F1
57      C      ROOM TEMP STIFFNESSES
58      SKU=2.02937*ETEMP(80.,IF )*(H3+(F15+4.*TR)/(1.-PR*PR)
59      1      *A2*B2)
60      C      ROOM TEMP FREQUENCY
61      FU=0.164*SQRT(SKU/SKM)
62      C      CALCULATE ROOM TEMP MEAN SQUARE STRESS RESPONSE
63      AR=3.*(B2/A2)* 2+3.*(A2/B2)* 2+2.
64      C      CONVERT DB TO PSI
65      SPL=2.91*10.*(PSL/20.-9.)
66      C      CALCULATE ROOM TEMP DYNAMIC STRESS AT X=0,Y=B2/2
67      SXU=0.36*B2*(B2*SQ. T(FU/DAMP)*SPL/(H*H*AR)
68      C      CALCULATE ROOM TEMP DYNAMIC STRESS AT X=A2/2,Y=0
69      SYU=1.30*A2*A2*SQRT(FU/DAMP)*SPL/(H*H*AR)
70      C      CONVERT STRESS FROM PSI TO KSI
71      SXU=SXU/1000.
72      SYU=SYU/1000.
73      C      CALCULATE ROOM TEMPERATURE LIFE
74      CALL SH(SXU,0.0,80.0,CTF1,IF )
75      CALL SH(SYU,0.0,80.0,CTF2,IF )
76      X1=A2/2.
77      Y1=0.0
78      X2=0.0
79      Y2=B2/2.
80      STEMP=0.0
81      C      PRINT ROOM TEMPERATURE RESPONSE
82      WRITE(6,400)
83      GO TO(201,202),IF
84      201 WRITE(6,405) NCASE
85      GO TO 203
86      202 WRITE(6,406) NCASE
87      203 WRITE(6,410) PSL,T
88      WRITE(6,415)
89      WRITE(6,416) FU

```

# TABLE III (CONT'D)

```

90      WRITE(6,420)
91      WRITE(6,425)
92      WRITE(6,430) X2,Y2,SX0,STEMP,CTF1
93      WRITE(6,430) X1,Y1,SY0,STEMP,CTF2
94      C
95      C      THERMAL STRESS EFFECTS
96      C
97      P2 =R(B2,A2,PR)
98      R21=R(B2,A1,PR)
99      R12=R(B1,A2,PR)
100     R11=R(B1,A1,PR)
101     F2S=F2*(F2+2.*(A1/A2)*2*F21+2.*(B1/B2)+2*F12
102     1      +4.*(A1/A2)*2*(B1/B2)+2*F11)
103     RST=R2+2.*A3*R12+2.*B3*R21+4.*A3*B3*R1
104     R0=(F1S+CTR)/F2S
105     C      CALCULATE CRITICAL TEMPERATURE RISE, TCR
106     TCALP=5.25*H*F2/(A2*B2*(1.+PR))
107     CAL CTMP(TCALP,TCR,IF)
108     C      *NOTE* TCA AND RS ARE BUCKLING TEMPERATURE
109     C      AND TEMPERATURE RATIO FOR AN EQUAL
110     C      SIZE SIMPLE PANEL. R9 IS TEMP RATIO
111     C      FOR NINE-BAY PANEL
112     TCA=TCR/R0
113     RS=T/TCA
114     R9=T/TCR
115     TACT=T+80.0
116     C      CALCULATE MATERIAL PROPERTIES AT TEMPERATURE
117     ES=ETEMP(TACT,IF)
118     ALP=ALPHA(TACT,IF)
119     D=0.0833*ES*H3/(1.-PR*PR)
120     C      CALCULATE RESPONSE FREQUENCY AT TEMPERATURE, T
121     SKT=D*F2S*R0/(A2*B2)
122     FQT=0.809*SQRT(SKT/SKM)
123     C      *NOTE* FQT=F0, ROOM TEMP FREQUENCY
124     STLIN=-ES*ALP*T/(1.-PR)/1000.0
125     IF (RS-R0)205,205,210
126     C      PRE-BUCKLED RESPONSE
127     205 FTEMP=FQT*(0.60+0.40*SQRT(1.-R9))
128     SXT=STLIN
129     SYT=STLIN
130     W0=0.0
131     GO TO 215
132     C      POST-BUCKLED RESPONSE
133     210 FTEMP=FQT*(0.60+0.44*SQRT(R9-1.))
134     C      CALCULATE PLATE BUCKLING AMPLITUDE, W0
135     W0=(3.37-0.20*R0)*H*SQRT(F2S*R0*(R9-1.)/RST)

```

TABLE III (CONT'D)

```

136 C      CALCULATE THERMAL STRESSES
137 C1=1./((A2*B2*(1.-PR*PR))
138 SXT=STLIN+0.81*ES*C1*((2.-PR*PR)*B2/A2+A2*PR/B2)
139 1      *W0*W0/1000.0
140 SYT=STLIN+1.36*ES*C1*(PR*B2/A2+(2.-PR*PR)*A2/B2)
141 1      *W0*W0/1000.0
142 C      CALCULATE DYNAMIC STRESS
143 215 CONTINUE
144 C2=SQRT(FTEMP/F0)
145 SX0=C2*SX0
146 SY0=C2*SY0
147 C      CALCULATE ELEVATED TEMPERATURE LIFE
148 CALL SN(SX0,SXT,TACT,CTF1,IFI)
149 CALL SN(SY0,SYT,TACT,CTF2,IFI)
150 C      PRINT ELEVATED TEMPERATURE RESPONSE
151 WRITE(6,435)
152 TA=TCR+80.0
153 WRITE(6,440) TCR
154 WRITE(6,445) W0
155 WRITE(6,446) FTEMP
156 WRITE(6,420)
157 WRITE(6,425)
158 WRITE(6,430) X2,Y2,SX0,SXT,CTF1
159 WRITE(6,430) X1,Y1,SY0,SYT,CTF2
160 GO TO 200
161 C      FORMAT STATEMENTS FOR OUTPUT DATA
162 40 FORMAT('1',25X,'DYNAMIC RESPONSE OF A',/,19X,
163 1'NINE-BAY STIFFENED PANEL EXPOSED TO',/,21X,
164 2'ACOUSTIC EXCITATION AND HEATING',/)
165 405 FORMAT(29X,'DATA CASE',I4,/,27X,'MATERIAL : TITANIUM')
166 406 FORMAT(29X,'DATA CASE',I4,/,27X,'MATERIAL : ALUMINUM')
167 410 FORMAT(5X,'EXCITATION SPECTRUM LEVEL = ',F4.0,1X,'DB',
168 13X,'TEMPERATURE INCREASE = ',F4.0,1X,'DEG. F',/)
169 415 FORMAT(24X,'ROOM TEMPERATURE RESPONSE',/)
170 416 FORMAT(20X,'FUNDAMENTAL FREQUENCY = ',F7.1,' HZ',/)
171 420 FORMAT(5X,'STRESS AT POINT',3X,'DYNAMIC STRESS',3X,
172 1'THERMAL STRESS',3X,'CYCLES TO FAILURE')
173 425 FORMAT(8X,'X',7X,'Y',1X,'KSI',14X,'KSI',/)
174 430 FORMAT(6X,'F5.2,3X,'F5.2,5X,'F8.3,9X,'F8.3,10X,'IPE',2,/)
175 435 FORMAT(/,2X,'ELEVATED TEMPERATURE RESPONSE',/)
176 440 FORMAT(10X,'BUCKLING TEMPERATURE = ',F8.2,' DEG. F'
177 1,' ABOVE ROOM TEMPERATURE',/)
178 445 FORMAT(18X,'BUCKLING AMPLITUDE = ',F8.4,
179 1' INCHES',/)
180 END

```

TABLE IV  
OUTPUT FORMAT FOR DYNAMIC ANALYSIS COMPUTER PROGRAM

DYNAMIC RESPONSE OF A  
NINE-BAY STIFFENED PANEL EXPOSED TO  
ACOUSTIC EXCITATION AND HEATING

DATA CASE 4

MATERIAL : ALUMINUM  
EXCITATION SPECTRUM LEVEL = 135. DB    TEMPERATURE INCREASE = 20. DEG. F

ROOM TEMPERATURE RESPONSE

FUNDAMENTAL FREQUENCY = 175.0 HZ

STRESS X	AT POINT Y	DYNAMIC STRESS KSI	THERMAL STRESS KSI	CYCLES TO FAILURE
.00	6.00	6.076	.0	6.43+00
3.00	.00	5.485	.0	1.03+07

ELEVATED TEMPERATURE RESPONSE

BUCKLING TEMPERATURE = 10.00 DEG. F ABOVE ROOM TEMPERATURE

BUCKLING AMPLITUDE = .215 INCHES

FUNDAMENTAL FREQUENCY = 417.5 HZ

STRESS X	AT POINT Y	DYNAMIC STRESS KSI	THERMAL STRESS KSI	CYCLES TO FAILURE
.00	6.0	9.371	-16.830	1.32+05
3.0	.0	8.460	-24.342	1.40+05

**TABLE V**  
**COMPUTER PROGRAM FOR CALCULATING ELASTIC MODULUS**

```

1      FUNCTION ETEMP(T, IFF)
2      C
3      C      THIS FUNCTION COMPUTES ELASTIC MODULUS FOR
4      C      ALUMINUM OR TITANIUM ALLOY AS A FUNCTION OF
5      C      TEMPERATURE
6      C
7      C      T - INPUT TEMPERATURE - DEG. F
8      C      IFF - ALLOY CODE
9      C      = 1 TITANIUM
10     C      = 2 ALUMINUM
11     C
12     C      GO TO (100,200), IFF
13     C      *****
14     C      MATERIAL 6AL-4V TITANIUM ANNEALED SHEET
15     C      REFERENCE MIL-HDBK-5B
16     C      TEMPERATURE LIMITATION 800 DEGREES F
17     C      RT<T<800 F
18     100 IF(800-T)180,190,150
19     190 ETEMP=(1.030-0.000375*T)*16.6E+06
20     RETURN
21     C      T>800 F
22     180 ETEMP=12.1E+06
23     WRITE(6,333)
24     RETURN
25     C      *****
26     C      MATERIAL 7075-T6 SHEET
27     C      REFERENCE MIL-HDBK-5B
28     C      TEMPERATURE LIMITATION 600 DEGREES F
29     C      RT<T<200 F
30     200 IF(200-T)220,210,210
31     210 ETEMP=(1.020-0.00030*T)*10.3E+06
32     RETURN
33     C      200<T<400 F
34     220 IF(400-T)240,230,230
35     230 ETEMP=(0.96-0.00070*(T-200))*10.3E+06
36     RETURN
37     C      400<T<600 F
38     240 IF(600-T)260,250,250
39     250 ETEMP=(0.82-0.0016*(T-400))*10.3E+06
40     RETURN
41     C      T>600 F
42     260 ETEMP=0.50*10.3E+06
43     WRITE(6,333)
44     333 FORMAT(/,5X,'UPPER TEMP LIMIT ON ELAST MODULUS',
45     1' EXCEEDED',/)
46     RETURN
47     END

```



TABLE VI

## COMPUTER PROGRAM FOR CALCULATING COEFFICIENT OF THERMAL EXPANSION

```

1      FUNCTION ALPHA(T,IFF)
2      C
3      C      THIS FUNCTION COMPUTES COEFFICIENT OF THERMAL
4      C      EXPANSION FOR ALUMINUM OR TITANIUM ALLOY AS A
5      C      FUNCTION OF TEMPERATURE
6      C
7      C      T - INPUT TEMPERATURE - DEG. F
8      C      IFF - ALLOY CODE
9      C          =1  TITANIUM
10     C          =2  ALUMINUM
11     C
12     GO TO (100,200), IFF
13     C *****
14     C      MATERIAL 6AL-4V TITANIUM SHEET
15     C      ANNEALED
16     C      REFERENCE MIL-HDBK-5B
17     C      TEMPERATURE LIMITATION 1000 DEGREES F
18     C      RT<T<200 F
19     100 IF(200-T)180,150,150
20     150 ALPHA=(4.45+0.00425*T)*1.0E-06
21     RETURN
22     C      200<T<400 F
23     180 IF(400-T)185,190,190
24     190 ALPHA=(4.86+0.0025*T)*1.0E-06
25     RETURN
26     C      400<T<1000 F
27     185 IF(1000-T)195,198,198
28     195 WRITE(6,500)
29     500 FORMAT(1,5X,'UPPER TEMP LIMIT ON COEFF OF EXPAN ',
30     1'EXCEEDED',/)
31     198 ALPHA=5.8E-06
32     RETURN
33     C *****
34     C      MATERIAL 7075-T6 SHEET
35     C      REFERENCE MIL-HDBK-5B
36     C      TEMPERATURE LIMITATION 600 DEGREES F
37     C      RT<T<100 F
38     200 IF(100-T)220,210,210
39     210 ALPHA=(12.4+0.0050*T)*1.0E-06
40     RETURN
41     C      100<T<300 F
42     220 IF(300-T)240,230,230
43     230 ALPHA=(12.9+0.00275*(T-100))*1.0E-06
44     RETURN
45     C      300<T<400 F
46     240 IF(400-T)260,250,250
47     250 ALPHA=(13.45+0.00150*(T-300))*1.0E-06
48     RETURN
49     C      T>400 F
50     260 ALPHA=13.6E-06
51     IF(600-T)280,270,270
52     280 WRITE(6,500)
53     270 RETURN
54     END

```

TABLE VII  
COMPUTER PROGRAM FOR CALCULATING FATIGUE LIFE

```

1      SUBROUTINE SN(SDYN,STEMP,TEMP,CTF,IFF)
2      C
3      C      THIS SUBROUTINE CALCULATES FATIGUE LIFE FOR
4      C      ALUMINUM OR TITANIUM ALLOY AS A FUNCTION OF
5      C      TEMPERATURE AND MEAN STRESS.  THIS SUBROUTINE
6      C      IS BASED ON COUPON FATIGUE TEST DATA AT ROOM
7      C      AND ELEVATED TEMPERATURE.
8      C
9      C      ROOM TEMPERATURE IS 80 DEG. F
10     C
11     C      SDYN - DYNAMIC STRESS - KSI (RMS)
12     C      STEMP - THERMAL (OR MEAN) STRESS - KSI
13     C      TEMP - TEMPERATURE - DEG. F
14     C      CTF - LIFE IN CYCLES TO FAILURE
15     C      IFF - ALLOY CODE
16     C           = 1 TITANIUM
17     C           = 2 ALUMINUM
18     C
19     C      GO TO(900,200), IFF
20     C      *****
21     C      MATERIAL 6AL-4V TITANIUM SHEET ANNEALED
22     C      TEMPERATURE LIMITATION 600 DEG. F
23     C
24     100  C1=12.58-0.00376*TEMP
25         C2=-5.40+0.00176*TEMP
26         ARF=C1+C2*ALOG10(SDYN-0.1*STEMP)
27         CTF=10.**ARF
28         RETURN
29     C      *****
30     C      MATERIAL 7075-T6 ALUMINUM SHEET
31     C      TEMPERATURE LIMITATION 300 DEG. F
32     C
33     200  C1=10.89-0.00584*TEMP
34         C2=-4.89+0.00347*TEMP
35         ARF=C1+C2*ALOG10(SDYN-0.1*STEMP)
36         CTF=10.**ARF
37         RETURN
38     END

```

TABLE VIII  
COMPUTER PROGRAM FOR CALCULATING SKIN BUCKLING TEMPERATURE

```

1      SUBROUTINE CTEMP(TCALP,TC,IFF)
2      C      THIS SUBROUTINE CALCULATES SKIN BUCKLING
3      C      TEMPERATURE FOR ALUMINUM OR TITANIUM ALLOY
4      C      STRUCTURAL PANELS.
5      C
6      C      TCALP - PRODUCT OF BUCKLING TEMPERATURE
7      C      AND ALPHA FROM CALLING PROGRAM
8      C      TC - BUCKLING TEMPERATURE - DEG F ABOVE
9      C      ROOM TEMPERATURE
10     C      IFF - ALLOY CODE
11     C      = 1 TITANIUM
12     C      = 2 ALUMINUM
13     C
14     TC=0.0
15     GO TO (100,200),IFF
16     C *****
17     C      MATERIAL 6AL-4V TITANIUM ANNEALED
18     C      TEMPERATURE LIMITATION 1000 DEG. F
19     C
20     100 C1=4.45E-06
21         C2=4.23E-09
22         I=1
23         1 C3=60.*C2+C1
24           TC=.5*SQRT((C3/C2)**2+4.*TCALP/C2)-.5*C3/C2
25           T=TC+80.
26           IF(T-260.) 50,50,2
27           2 IF(T-360.) 3,3,5
28           3 C1=4.9E-06
29             C2=2.5E-09
30             I=I+1
31             IF(I-2) 1,1,50
32             5 C1=5.80E-06
33               TC=TCALP/C1
34             50 TC=TC
35             RETURN

```

TABLE VIII (CONT)

```

36 C *****
37 C MATERIAL 7075-T6 ALUMINUM ALLOY
38 C TEMPERATURE LIMITATION 600 DEG. F
39 C
40 200 F1=12.4E-06
41 F2=5.0E-09
42 II=1
43 201 F3=80.*F2+F1
44 TC=0.50*SQRT((P3/F2)**2+4.0*TCALP/F2)+0.50*F3/F2
45 T=TC+80.
46 IF(T-100.)500,500,202
47 202 IF(T-300.)203,203,204
48 203 F1=12.625E-06
49 F2=0.75E-09
50 II=II+1
51 IF(II-2)201,201,500
52 204 IF(T-400.)205,205,206
53 205 F1=13.0E-06
54 F2=1.5E-09
55 II=II+1
56 IF(II-2)201,201,500
57 206 F1=13.6E-06
58 TC=TCALP/F1
59 500 TC=TC
60 RETURN
61 END

```

TABLE IX  
COMPUTER PROGRAM FOR CALCULATING SECTION PROPERTIES

```

1      SUBROUTINE PROP(OPT,B,H,T,A,RJ,GAMAT,XIP)
2      C
3      C      SECTION PROPERTIES
4      C      IF OPT = 0 ZEE SECTION
5      C      IF OPT = 1 CHANNEL SECTION
6      C      H= STRINGER HEIGHT, CL TO CL
7      C      B= FLANGE WIDTH
8      C      T= STRINGER THICKNESS
9      C      REFERENCE: AFFDL-TR-71-107
10     C      WC = WARPING CONSTANT ABOUT SHEAR CENTER
11     C      GAMAT = WARPING CONSTANT ABOUT ATTACH POINT
12     C      A = CROSS SECTIONAL AREA
13     C      RJ= ST. VENANTS TORSION CONSTANT = J
14     C      XIP = POLAR MOMENT OF INERTIA ABOUT ATTACH POINT
15     IF(OPT) 1,1,2
16     C      ZEE STIFFENER
17     1  A=T*(H+2.*B)
18       O=H**2*(6.*B+H)
19       D1=T**2*(3.*H+2.*B)
20       XXI=(T*(O+D1))/12.
21       D2=2.*B+T
22       D3=2.*B-T
23       XZI=-(T*H*D2*D3)/8.
24       ZZI=(T/12.)*(8.*B**3+H*T**2)
25       RJ=(T**3/3.)*(2.*B+H)
26       WC=T*B**3*H**2*(B+2.*H)/(12.*(2.*B+H))
27       SX=B/2.
28       SZ=-H/2.
29       D4=SX**2+SZ**2
30       D5=D4*A
31       XIP=(XXI+ZZI+D5)
32       GAMAT=WC+(SZ**2)*ZZI-2.*SX*SZ*XZI+(SX**2)*XXI
33       RETURN
34     C      CHANNEL SECTION
35     2  F=2.*B+H
36       XBAR=B**2/F
37       F1=6.*B+H
38       E=3.*B**2/F1
39       CX=E+XBAR
40       SX=E+(B/2.)
41       SZ=-H/2.
42       EX=CX-SX
43       A=T*(H+2.*B)
44       F2=3.*H+2.*B
45       XXI=T*(H**2*F1+T**2*F2)/12.
46       F3=12.*H*XBAR**2+8.*B**3
47       F4=B-XBAR
48       F5=B-2.*XBAR
49       ZZI=T*(F3-24.*XBAR*B*F4+12.*B*F5*T+6.*F4*T**2+T**3)/12.
50       RJ=T**3*F/3.
51       F6=3.*B+2.*H
52       WC=T*B**3*H**2*F6/(12.*F1)
53       GAMAT=WC+(SZ**2)*ZZI+(SX**2)*XXI
54       F7=(EX**2+SZ**2)*A
55       XIP=(XXI+ZZI+F7)
56       RETURN
57     END

```

## VI - LIMITATIONS

Application of these design procedures should be tempered with a thorough understanding of their limitations. Certain of the initial assumptions stated during the analytical development were negated by derivation of the empirical relations. However, the limits in the physical and environmental parameters of the experimental program then apply to these design criteria, as itemized below.

### A. Physical Constraints

The bounds of the test specimen dimensions were used to establish these limitations. These should be treated only as a guide, since the equations and nomographs are normally valid beyond these limits. Individual judgment must be applied in unusual cases where the constraints are drastically exceeded, particularly in case of the design charts. The size guidelines are:

- o Panel bay width:  $a = 5$  to 9 inches
- o Panel bay aspect ratio:  $b/a = 1.5$  to 3
- o Panel skin thickness:  $h = 0.024$  to 0.063 inch

### B. Environmental Constraints

The acoustic environment generally has no restrictions with regard to the applicability of the design criteria. Spectrum levels below 120 dB will normally result in low dynamic stresses and a long fatigue life. The higher noise levels will generally result in non-linear response, but these effects are included in the design criteria, since many of the test specimens exhibited a high degree of nonlinearity.

The thermal environment must be nearly uniform over the surface of a panel bay. The skin temperature is limited to the maximum temperature for which the alloys retain significant structural properties. These limiting temperatures are, for the alloys considered:

- o 7075-T6 aluminum - 300°F maximum
- o 6Al-4V annealed titanium - 600°F maximum

The design life criteria are based only on specific nominal temperatures, requiring the use of interpolation for intermediate temperatures. Extrapolation beyond the temperature limits may be permissible to some extent if care is exercised and the further temperature degradation effects are included.

Probably the most important restriction to the design method is in the estimation of the ambient temperature and the state of the structure at this temperature. All thermal response relations are referenced to the ambient temperature and the assumption that a state of stress equilibrium exists (i.e., no mean stresses). It is impractical at this stage to give guidelines for estimating the ambient temperature state because it is dependent on factors

such as the length of time at a uniform temperature and external constraints. It should be noted that a change in ambient temperature over a short interval constitutes a temperature change as far as the response relations are concerned.

### C. External Constraints

The external constraints imposed on the test panels precluded significant thermal expansion of the substructure. This is considered representative of structural applications in the direct flow path of engine exhausts or other heat sources, where only localized areas of the structure are heated. The criteria can also be applied to design applications involving gradual heating of an entire structural area (i.e., supersonic aircraft), where all structure expands at about the same rate. This corresponds to a relaxation of the external constraints from those considered here. In this case, the thermal buckling amplitudes and stresses given by the relations delineated herein will result in a conservative design.

It should be noted that the empirical results presented herein are applicable only for the case of simultaneous application of heat and noise. Alternate application of these environments, wherein significant thermal stress cycles are incurred, were not considered in this program.

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## APPENDIX I

### GENERAL EXPRESSIONS FOR STIFFENER GEOMETRIC PROPERTIES

(From Reference 3)

The geometric parameters defined here are developed in terms of a centroidal ( $x, y, z$ ) coordinate system. General expressions for the cross-sectional area, the area moments, torsion constant, and warping constants are presented for zee and channel cross-section shapes. These parameters are defined as follows:

$\bar{x}$  - the location of the centroid

$e$  - the location of the shear center

$A$  - the cross-sectional area

$$I_{xx} = \int_A z^2 dA ; I_{xz} = \int_A xz dA ; I_{zz} = \int_A x^2 dA$$

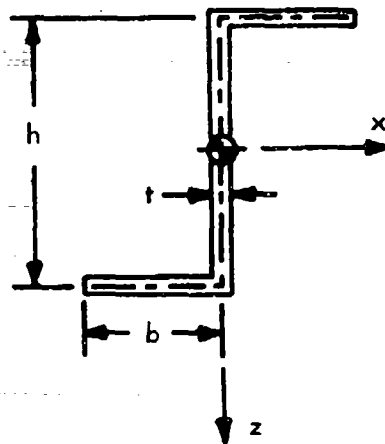
$$J = \int_A \left[ (\phi_{,x} - z)^2 + (\phi_{,z} + x)^2 \right] dA \quad (\text{St. Venant's Torsion Constant})$$

$$R_{Ez} = \int_A x\phi dA \quad R_{Ex} = \int_A z\phi dA \quad \Gamma_e = \int_A \phi^2 dA$$

where  $\phi(x, z)$  is the warping function for the cross-section with the pole taken as the shear center.

---

NOTE: The Symbols used in this appendix are applicable only for this section. Since they are defined here or in the following pages, they are not included in the List of Symbols.



$$A = t(h + 2b)$$

$$I_{xx} = \frac{t}{12} [h^2(6b + h) + t^2(3h + 2b)]$$

$$I_{xz} = -\frac{th}{8} (2b + t)(2b - t)$$

$$I_{zz} = \frac{t}{12} [8b^3 + ht^2]$$

$$J = \frac{t^3}{3} [2b + h]$$

$$\Gamma_e = \frac{tb^3h^2(b + 2h)}{12(2b + h)}$$

FIGURE 1 - 1 GEOMETRIC PROPERTIES - ZEE SECTION  
(FROM REFERENCE 3)

$$\bar{x} = b^2 / (2b + h)$$

$$e = 3b^2 / (6b + h)$$

$$A = t(h + 2b)$$

$$I_{xx} = \frac{t}{12} [h^2(6b + h) + t^2(3h + 2b)]$$

$$I_{xz} = 0$$

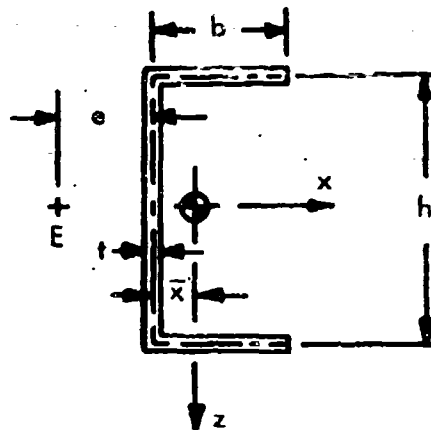
$$I_{zz} = \frac{t}{12} [12h\bar{x}^2 + 8b^3 - 24\bar{x}b(b - \bar{x}) + 12b(b - 2\bar{x})t + 6(b - \bar{x})t^2 + t^3]$$

$$J = \frac{1}{3} t^3 (2b + h)$$

$$\Gamma_o = \frac{tb^3 h^2 (3b + 2h)}{12(6b + h)}$$

$$R_{Ez} = 0$$

$$R_{Ex} = 0$$



NOTE: For warping constants, the pole is taken at the shear center.

FIGURE 1- 2 GEOMETRIC PROPERTIES - CHANNEL SECTION  
(FROM REFERENCE 3)

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13. ABSTRACT An analytical and experimental program was conducted to develop acoustic fatigue design criteria for aircraft structures subjected to intense noise in a high temperature environment. Equations for the dynamic response of a buckled panel were formulated for simply supported boundary conditions using large deflection plate theory. Random amplitude acoustic fatigue testing of representative aircraft structure was accomplished at temperatures up to 600°F to provide data for correlation with the analytical results. Empirical design criteria were developed in the form of equations and nomographs for predicting the thermal and dynamic response of aircraft structures subjected to combined environments. The empirical design criteria are presented in handbook format for design use; examples and computer programs are also presented.		

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